Midterm 01

Math 32a @ $\mathscr{U}CLA$ (Fall 2021)

Assigned: October 22, 2021.

Last Name: ______________________

First Name: ______________________

Student ID ______________________

Instructions/Admonishment

1. SHOW ALL WORK.
   A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. You may not use books, calculators, notes, or any other material to help you. Please make sure **your phone is silenced** and stowed where you cannot see it.
2. Duration: 50 min.
3. The following is my own work, without the aid of any other person.
   Signature: ______________________

Please do not write below this line.

Problem	Points	Scores
1	25	
2	25	17 +3
3	25	33
4	25	33
Total	100	83+3=86

Problem 2 ***A point at an intersection.***

Let l be the line whose parametric equation is defined by $\mathbf{r}(t) = \langle 1+t, 1-2t, 3+5t \rangle$. Let $\mathscr{P}$ be the plane $2x - y + 3z - 48 = 0$.

(i) Find the point of intersection Q of the line l and the plane $\mathscr{P}$.

(ii) Find an equation of the plane $\mathscr{R}$ that passes through the point Q and is orthogonal to l.

(i)

$r(t) = \langle 1+t, 1-2t, 3+5t \rangle$

$2(1+t) - (1-2t) + 3(3+5t) - 48 = 0$

$2 + 2t - 1 + 2t + 9 + 15t - 48 = 0$

$19t = 48 + 1 - 9 - 2$

$19t = 38$

$t = 2$

$Q = r(2) = \langle 1+2, 1-2(2), 3+5(2) \rangle$

$\boxed{Q = \langle 3, -3, 13 \rangle}$

(ii) $r(t) = \langle 1, 1, 3 \rangle + t \langle 1, -2, 5 \rangle$

direction vector

$\perp l \rightarrow \langle 1, -2, 5 \rangle \cdot \vec{n} = 0$

$1(n_1) + (-2)(n_2) + 5(n_3) = 0$

$1(-1) + (-2)(2) + 5(1) = 0$

$-1 - 4 + 5 = 0$

$0 = 0$ ✓

$\vec{n} = \langle -1, 2, 1 \rangle$

$-x + 2y + z = d$

$Q = \langle x_0, y_0, z_0 \rangle = \langle 3, -3, 13 \rangle$

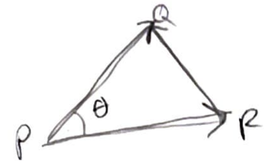
$d = ax_0 + by_0 + cz_0$

$d = (-1)(3) + 2(-3) + 1(13)$

$d = -3 - 6 + 13$

$d = 4$

$\boxed{-x + 2y + z = 4}$



Problem 3 *The triangle.*
Let $P = (3,1,3)$, $Q = (2,0,-1)$, and $R = (-1,1,-3)$.

(i) Does PQR form a right triangle? Find the area of the triangle PQR.

(ii) Find the equation of the plane determined by the points P, Q and R.

$\vec{PQ} = \langle 2-3, 0-1, -1-3 \rangle$

$\vec{PQ} = \langle -1,-1,-4 \rangle$

$\vec{PR} = \langle -1-3, 1-1, -3-3 \rangle$

$\vec{PR} = \langle -4, 0, -6 \rangle$ ✓

$\vec{RQ} = \langle 2-(-1), 0-1, -1-(-3) \rangle$

$\vec{RQ} = \langle 3,-1,2 \rangle$

(i) if PQR is a right triangle, there is 1 90° angle (orthogonal)

$\vec{PQ} \cdot \vec{PR} = 0$

$(-1)(-4) + (-1)(0) + (-4)(-6) = 0$

$4 + 24 = 0$ ✗ $28 \neq 0$

$\vec{PR} \cdot \vec{RQ} = 0$

$(-4)(3) + 0(-1) + (-6)(2) = 0$

$-12 - 12 = 0$ ✗ $-24 \neq 0$

✓ $\vec{PQ} \cdot \vec{RQ} = 0$

$(-1)(3) + (-1)(-1) + (-4)(2) = 0$

$-3 + 1 - 8 = 0 \quad -10 \neq 0$ ✗

$\boxed{\text{no, } \vec{PQR} \text{ doesn't form a right } \Delta.}$

$\vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ -1 & -1 & -4 \\ -4 & 0 & -6 \end{vmatrix}$

$= \langle 6-0, -(6-16), 0-4 \rangle$

$= \langle 6, 10, -4 \rangle$

$\text{Area}\triangle PQR = \dfrac{\|\vec{PQ} \times \vec{PR}\|}{2}$

$= \dfrac{\sqrt{6^2+10^2+(-4)^2}}{2}$ ✓

$36 + 16 = 52$

$\boxed{A = \dfrac{\sqrt{152}}{2}} \Rightarrow \dfrac{\cancel{2}\sqrt{38}}{\cancel{2}}$

$\boxed{A = \sqrt{38}}$

(ii) plane needs $\vec{n} = \vec{PQ} \times \vec{PR}$

✓ $\vec{n} = \langle 6, 10, -4 \rangle$

$6x + 10y - 4z = d$

$P = (3,1,3)$

$d = 6(3) + 10(1) - 4(3)$

$d = 18 + 10 - 12$

$d = 16$

$\dfrac{6x + 10y - 4z}{2} = \dfrac{16}{2}$

$\boxed{3x + 5y - 2z = 8}$ ✓

verify: by showing Q and R are in the plane

$Q = (2,0,-1)$

$3(2) + 5(0) - 2(-1) = 8$

$6 + 2 = 8$ ✓

$R = (-1,1,-3)$

$3(-1) + 5(1) - 2(-3) = 8$

$-3 + 5 + 6 = 8$

$11 - 3 = 8$ ✓

Problem 4

With distance measured in $\boxed{\text{feets,}}$ a particle position is given by $\mathbf{r}(t) = \langle 1 - 2\sin t, 1 - 2\cos t, 0 \rangle$.

(i) Find the speed at time $t = 2$ seconds. $\frac{ds}{dt} = \|r'(t)\|$

(ii) Find an arc length parametrization of the particle position.

(i) $r'(t) = \langle -2\cos t, 2\sin t, 0 \rangle$

speed $= \frac{ds}{dt} = \|r'(t)\| = \sqrt{(-2\cos t)^2 + (2\sin t)^2 + (0)^2}$

$= \sqrt{4\cos^2 t + 4\sin^2 t + 0}$

$= \sqrt{4(\cos^2 t + \sin^2 t)}$

$= \sqrt{4} = 2$ ✓

we already found $\|r'(t)\|$

i) $\boxed{\text{speed} = 2\frac{ft}{s}}$

(ii)

$\|r'(t)\| = 2$ $\quad S = \int_0^t \|r'(u)\| du$

$S = \int_0^t 2 \, du$

$S = 2u \Big|_0^t$

$S = 2t - 2(0)$

$S = 2t$

$t = \frac{s}{2}$

to verify $r(s)$ is the arc length parametrization:

$\|r'(s)\| = 1$

$r'(s) = \langle -\cos(\frac{s}{2}), \sin(\frac{s}{2}), 0 \rangle$

$\|r'(s)\| = \sqrt{[-\cos(\frac{s}{2})]^2 + [\sin(\frac{s}{2})]^2 + 0^2}$

$\|r'(s)\| = \sqrt{\cos^2(\frac{s}{2}) + \sin^2(\frac{s}{2})}$

$\|r'(s)\| = \sqrt{1} = 1$ ✓

ii) $\boxed{r(s) = \langle 1 - 2\sin(\frac{s}{2}), 1 - 2\cos(\frac{s}{2}), 0 \rangle}$ ✓