TEST 1

MATH 32A @ UCLA (FALL 2020)

Assigned: October 30, 2020.

Instructions/Admonishment

1. SHOW ALL WORK.
   A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit.

2. Duration: 24 hours.

3. The following is my own work, without the aid of any other person.
   Signature: _____

Exercise 1 TRUE OR FALSE.

State whether the folowing statements are $TRUE$ or $FALSE$. Explain your choice.

(i) There exists nonzero vectors $\mathbf{a}$ and $\mathbf{b}$ such that $|\mathbf{a} \cdot \mathbf{b}| = ||\mathbf{a} \times \mathbf{b}||$

(ii) Given two parametrizations $\mathbf{r}_1(t) = \mathbf{w}_1 + t\mathbf{v}_1$ and $\mathbf{r}_2(t) = \mathbf{w}_2 + t\mathbf{v}_2$ of the same line, if $\mathbf{v}_1 = \mathbf{v}_2$, then $\mathbf{w}_1 = \mathbf{w}_2$.

Exercise 2 EXAMLPLES.
Give one example of:

(i) A curve in 3 Space with constant curvature. Give a parametric equation of such curve.

(ii) Two planes that are neither parallel nor perpendicular. Give an equation for each of such plane and explain your example.

Exercise 3 PROJECTION VECTOR.

Let $P = (9, 2, 1)$, $Q = (5, 5, 2)$, and $R = (7, -2, 4)$. Let $\mathbf{a} = \overrightarrow{PQ}$ and $\mathbf{b} = \overrightarrow{PR}$.

(i) Find $\mathbf{a}_{||\mathbf{b}}$, the projection vector of $\mathbf{a}$ along $\mathbf{b}$.

(ii) Do P, Q, and R lie on the same line? Explain using the projection you found in part (i).

Exercise 4 INTERSECTION OF A LINE AND A PLANE.

Write the equation of a line going though the point $P = (2, 0, -1)$ perpendicalur to the plane determined by the lines $\mathbf{r}_1(t) = \langle -5 + 2t, 3 - t, 1 - 2t \rangle$ and $\mathbf{r}_2(t) = \langle -5 - t, 3, 1 + t \rangle$

(i) Solve the problem.

(ii) Explain in words your solution process.

Exercise 5 PLANE DETERMINED BY A TRIANGLE.
Let $P = (1, 0, -1)$, $Q = (2, 3, 1)$, and $R = (0, 1, 2)$.

(i) Does PQR form a right triangle? Find the area the triangle PQR.

(ii) Find the equation of the plane determined by the points P, Q and R.

Exercise 6 CURVATURE OF A CURVE.

Consider the curve with the vector equation $\mathbf{r}(t) = \langle 3\sin(t), 4t, -3\cos(t)\rangle$.

(i) For what value(s) of t does $\mathbf{r}(t)$ lie on the sphere $x^2 + y^2 + z^2 = 41$?

(ii) Find the unit tangent vector $\mathbf{T}(t)$ and the curvature $\kappa(t)$.