FINAL EXAM

MATH 32A @ UCLA (FALL 2020)

Assigned: December 16, 2020.

Instructions/Admonishment

1. SHOW ALL WORK.
   A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit.

2. Duration: 24 hours.

3. The following is my own work, without the aid of any other person.
   Signature: ____

Exercise 1 INTERSECTION OF A LINE AND A PLANE.

(i) Find the equation of the plane that passes through the points $A = (2, 1, 1)$, $B = (-1, -1, 10)$ and $C = (1, 3, -4)$

(ii) Find an equation for the line through B that is perpendicular to the plane in part (i).

(iii) A second plane passes through $(2, 0, 4)$ and has normal vector $\mathbf{n} = \left\langle 2, -4, -3 \right\rangle$. Find the parametric equations (in scalar form) for the line of intersection of the two planes.

Exercise 2 THE TRIANGLE.

Let $A_1 = (1, 0, 1)$, $A_2 = (-1, 1, 1)$ and $A_3 = (0, 0, 2)$. The set of (x, y) such that A_1, A_2, A_3 and $(x, y, 0)$ are on the same plane is a line in the xy-plane. Find the intersection of such line with the x-axis.

Exercise 3 CURVE IN THREE-SPACE.

Consider the path of a particle moving in the Three Space $\mathbf{r}(t) = \left\langle \sin 2t, \cos 2t, 3t - 1 \right\rangle$, for $1 \leq t \leq 3$.

(i) Compute the total length of the path.

(ii) Find an equation of the tangent line to the path at $t = 2$.

(iii) Write the acceleration vector $\mathbf{a}$ as the sum of the tangential and normal components at $t = 2$.

Exercise 4 LIMIT AND CONTINUITY.

(i) Compute the following limit or explain why it does not exist.
$\lim_{(x,y)\to(0,0)} \left(\frac{x^2+\sin^2(y)}{x^2+2y^2}\right).$

(ii) Where is the function

$$f(x,y) = \begin{cases} \frac{x^2\sin(x)}{x^2+y^2} & if\ (x,y) \neq (0,0). \\ 0 & if\ (x,y) = (0,0). \end{cases}$$

continuous ?

Exercise 5 DIFFERENTIABILITY .

Consider the function

$$f(x,y) = \begin{cases} \dfrac{2xy^2}{x^2+y^4} & if\ (x,y) \neq (0,0). \\ 0 & if\ (x,y) = (0,0). \end{cases}$$

(i) Give the reason why $f(x,y)$ is not differentiable at $(0,0)$.

(ii) Use the limit definition of partial derivative to compute $f_x(0,0)$ and $f_y(0,0)$. What is your conclusion?

Exercise 6 THE MARTIAN ROVER.

The elevation of a region on Mars is given by $f(x, y) = x^2 + 2y^2 + xy - 2y$. A Mars rover is currently at the point $(x, y, z) = (0, 1, 0)$.

(i) Compute $\nabla f(x, y)$.

(ii) The rover locates an interesting rock at the bottom of the neaby valley. In what direction (in the xy-plane) should the rover move in order to decrease its elevation most quickly?

(iii) Compute $\frac{d}{dt} f(\mathbf{r}(t))$ if the rover moves along the path $\mathbf{r}(t) = \left\langle t - t^2, 1 - 2t \right\rangle$, for $t \geq 0$.

Problem 7 CHAIN RULE

Consider the surface $f(x, y, z) = x^2y + y^2z$. Suppose $x = s + t$, $y = st$, and $z = 2s - t$.

(i) Use the **chain rule** to compute f_s in terms of s and t.

(ii) Use (i) to compute $f_s\,(s, t)$ at $(2, -1)$.

Exercise 8 TANGENT PLANE.

Consider the ellipsoid $F(x, y, z) = \dfrac{x^2}{9} + \dfrac{y^2}{25} + z^2 - 1 = 0.$

(i) Find an equation of the plane tangent of the ellipsoid at $(0, 4, \frac{3}{5})$

(ii) At what points on the ellipsoid is the tangent plane horizontal?

Exercise 9 OPTIMIZATION.

Find all absolute maxima and minima of $f(x, y) = e^{xy}$ in the region $9x^2 + y^2 \leq 1$. For finding critical points around the boundary of the region, use either Lagrange multipliers, or a parametrization of the boundary (i.e. $x(t) = 3\cos t, y(t) = \sin t$, for $0 \leq t \leq 2\pi$), or any other strategy.