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Math 32A

April 20, 2016

Midterm 1

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UID [redacted]

Section 3A (Jacob, Tues 2 pm)

Write everything that you want graded on these pages and clearly indicate your answers. If you need more space, use the back of these pages and clearly indicate where the continuation may be found. Write as legibly as possible since I will ignore anything I cannot read (or find). You must show your work to obtain full credit. Points may be deducted if you do not justify your final answer. No aids such as calculators, notes, and textbooks are allowed.

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1. (20 pts) Consider the vectors $\mathbf{u} = (3, 0, 1)$, $\mathbf{v} = (0, 2, 1)$, and $\mathbf{w} = (0, 2, 2)$.

(a) (10 pts) Find the area of the parallelogram spanned by **u** and **v**.

$$A(\square) = \|u \times v\|$$

$$u \times v = \begin{vmatrix} i & j & k \\ 3 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} = i\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} - j\begin{vmatrix} 3 & 1 \\ 0 & 1 \end{vmatrix} + k\begin{vmatrix} 3 & 0 \\ 0 & 2 \end{vmatrix} = i(0-2) - j(3-0) + k(6-0)$$

$$= (-2, -3, 6)$$

$$\|u \times v\| = \sqrt{4+9+36} = \sqrt{49} = \boxed{7}$$

✓

(b) (10 pts) Find the volume of the parallelepiped spanned by **u**, **v**, and **w**.

$$\begin{vmatrix} 3 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 2 & 2 \end{vmatrix} = 3\begin{vmatrix} 2 & 1 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 0 & 2 \\ 0 & 2 \end{vmatrix} = 3(4-2) = 6 \checkmark$$

$$u \cdot (v \times w) = (u \times v) \cdot w$$

$$(-2, -3, 6) \cdot (0, 2, 2) = 0 - 6 + 12 = \boxed{6}$$

✓

2. (20 pts) Give equations for two distinct planes (besides the xy-plane) in $\mathbf{R}^3$ whose trace in the xy-plane has equation $3x + 4y = 8$.

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$$\boxed{\begin{aligned} 3x + 4y + 5z &= 8 \\ 3x + 4y + 6z &= 8 \end{aligned}}$$

xy plane $\rightarrow z = 0$

if $z = 0$, $3x + 4y = 8$ ✓

3. (25 pts) Let $\mathcal{P}$ be the plane given by $2x + 3y - z = 15$ and let P_0 be the point $(1, 2, 4)$.

(a) (10 pts) Find parametric equations for the line which is perpendicular to $\mathcal{P}$ and contains the point P_0.

n

10

$n = (2, 3, -1)$

$\vec{r}(t) = (2, 3, -1)t + (1, 2, 4)$

$$\boxed{\begin{aligned} x(t) &= 1 + 2t \\ y(t) &= 2 + 3t \\ z(t) &= 4 - t \end{aligned}}$$

(b) (15 pts) Find the distance from the point P_0 to the plane $\mathcal{P}$.

answer is on the back of this page

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$\overrightarrow{P_0Q} = \lambda n$

$= \lambda(2, 3, -1)$

$(1, 2, 4) - (x, y, z) = \lambda(2, 3, -1)$

$1 - x = 2\lambda$ $\quad x = 1 - 2\lambda$ $\quad 2(1-2\lambda) + 3(2 - 3\lambda) - (4 + \lambda) = 15$

$2 - y = 3\lambda$ $\quad y = 2 - 3\lambda$ $\quad 2 - 4\lambda + 6 - 9\lambda - 4 - \lambda = 15$

$4 - z = -\lambda$ $\quad z = 4 + \lambda$ $\quad -14\lambda + 4 = 15$

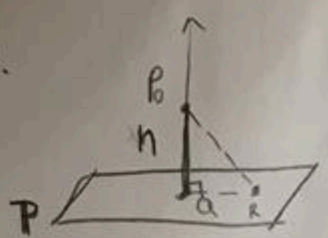
$\lambda = -\frac{11}{14}$

$\|\lambda\| = \frac{11}{14}$

$\|\overrightarrow{P_0Q}\| = \|(\frac{22}{14}, \frac{33}{14}, -\frac{11}{14})\| = \frac{\sqrt{22^2 + 33^2 + 11^2}}{14}$

$= \frac{11\sqrt{4 + 9 + 1}}{14} = \boxed{\frac{11\sqrt{14}}{14}}$

3) b.



$n = (2,3,-1)$

$\|n\| = \sqrt{4+9+1}$

$= \sqrt{14}$

R = any point on the plane

$R = (0,0,-15)$

$P_0 = (1,2,4)$

$\overrightarrow{P_0R} = (0,0,-15) - (1,2,4)$

$= (-1,-2,-19)$

$$\overrightarrow{P_0Q} = \overrightarrow{P_0R}_{\parallel n} = \left(\frac{\overrightarrow{P_0R}\cdot n}{\|n\|}\right)\frac{n}{\|n\|}$$

$$= \frac{(\overrightarrow{P_0R}\cdot n)\,n}{14}$$

$$= \frac{(-1,-2,-19)\cdot(2,3-1)}{14}(2,3,-$$

$$= \frac{-2-6+19}{14}(2,3,-1)$$

$$= \frac{11}{14}(2,3,-1)$$

$$\|\overrightarrow{P_0Q}\| = \frac{11}{14}\|n\|$$

$$= \frac{11}{14}\cdot\sqrt{14}$$

$$= \boxed{\frac{11\sqrt{14}}{14}}$$

$$\frac{y^2}{(\sqrt{2})^2} + \frac{z^2}{(\sqrt{3})^2} = 1$$

4. (10 pts) Find a parametrization of the ellipse $\frac{y^2}{2} + \frac{z^2}{3} = 1$ which has been translated to have center $(7, 1, 8)$.

$r(t) = (0, \sqrt{2}\cos t, \sqrt{3}\sin t) + (7, 1, 8)$

$\boxed{r(t) = (7, \sqrt{2}\cos t + 1, \sqrt{3}\sin t + 8)}$ 10

$$\frac{(\sqrt{2}\cos t)^2}{2} + \frac{(\sqrt{3}\sin t)^2}{3} = 1$$

$\frac{2\cos^2 t}{2} + \frac{3\sin^2 t}{3} = 1$ $\quad \cos^2 t + \sin^2 t = 1$ ✓

5. (25 pts) Consider the curve $\mathbf{r}(t) = (t^2, 2t, \ln(t))$.

(a) (10 pts) Find the tangent line to $\mathbf{r}(t)$ at $t = 1$.

$p = (1^2, 2(1), \ln(1)) = (1, 2, 0)$

$v = r'(1) = (2t, 2, \frac{1}{t})|_{t=1} = (2, 2, 1)$

tangent line:

$r(t) = (1, 2, 0) + t(2, 2, 1)$ 10

$\boxed{r(t) = (1 + 2t, 2 + 2t, t)}$

(b) (15 pts) Find the arc length of $\mathbf{r}(t)$ from $t = 1$ to $t = e$. $(t^{-1} + 2t)^2 = t^{-2} + 4\frac{t}{t} + 4t^2$

$= t^{-2} + 4 + 4t^2$

$S = \int_1^e \|r'(t)\|\, dt = ?$

$r'(t) = (2t, 2, \frac{1}{t})$

$\|r'(t)\| = \sqrt{4t^2 + 4 + \frac{1}{t^2}} = \sqrt{(t^{-1} + 2t)^2} = \frac{1}{t} + 2t$

$S = \int_1^e (\frac{1}{t} + 2t)\, dt = (\ln t + t^2)|_1^e$

$= (\ln e + e^2) - (\ln 1 + 1)$

$= 1 + e^2 - 1$ 15

$= \boxed{e^2}$

$\frac{d}{dt}(\ln t + t^2) = \frac{1}{t} + 2t$ ✓