MATH 32A: CALCULUS OF SEVERAL VARIABLES
WINTER 2017 - LECTURE 3
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MIDTERM 1

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By signing below, you confirm that you did not cheat on this exam. No exam booklet without a signature will be graded.

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## INSTRUCTIONS

- Please do not open this booklet until you are told to do so.
- You are only to use items necessary for writing. No other devices of any kind are permitted.
- No books or notes.
- If you have a question at any time during the exam, please raise your hand.
- You will receive points only for work written on the numbered pages. Please use the reverse side as scratch paper.
- Make sure to write legibly. Illegible work will not be graded.
- Make sure to show all your work and justify your answers fully.
- If you finish early, please wait in your seat until the time is called.

## SCORE

1. 9
2. 10
3. 7
4. 10
5. 10

TOTAL 46

1. (10 pts) Find an equation of the tangent line at the point $P = (10, 15)$ for

$$c(t) = (t^2 + 1, t^3 - 4t).$$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$x = t^2 + 1$

$10 = t^2 + 1$

$9 = t^2$

$t = +3$

or

$\cancel{t = -3}$

$y = t^3 - 4t$

$y = 3^3 - 12$

$= 27 - 12$

$= 15$

$\boxed{t = 3}$ parameterizes point P.

$c(3) = (10, 15)$

why not?

$-1$

$\frac{dy}{dt} = 3t^2 - 4$

$\frac{dx}{dt} = 2t$

$\frac{dy/dt}{dx/dt} = \frac{3t^2 - 4}{2t}$

$\left.\frac{dy}{dx}\right|_{t=3} = \frac{3(3)^2 - 4}{2(3)}$

$= \frac{3(9) - 4}{6}$

$= \frac{27 - 4}{6}$

$= \frac{23}{6}$

$y - y_1 = m(x - x_1)$

$y - 15 = \frac{23}{6}(x - 10)$

$y - 15 = \frac{23}{6}x - \frac{230}{6}$

$\boxed{y = \frac{23}{6}x - \frac{70}{3}}$

$15 \times 6 = 90$

$-\frac{230}{6} + \frac{90}{6}$

$\frac{-140}{6} = \frac{-70}{3}$

$230 - 90 = 140$

$140 \div 6$

$140 \div 3$

2. (10 pts) Determine whether the following two lines intersect:

$$\mathbf{r}_1(t) = \langle 1,0,1\rangle + t\langle 3,3,5\rangle, \quad x = 1+3t \quad y = 3t \quad z = 1+5t$$
$$\mathbf{r}_2(s) = \langle 3,6,1\rangle + s\langle 4,-2,7\rangle. \quad x = 3+4s \quad y = 6-2s \quad z = 1+7s$$

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set the components equal to each other.

① $1+3t = 3+4s$
② $3t = 6-2s$
③ $1+5t = 1+7s$

using ① & ② $\quad 1+3t = 3+4s \Rightarrow 3t = 2+4s$ ①
$\quad 3t = 6-2s \Rightarrow -(3t = 6-2s)$ ②

$$-3t = -6+2s$$
$$3t = 2+4s$$
$$0 = -4+6s$$
$$4 = 6s$$
$$s = \frac{2}{3} \rightarrow \text{② find } t$$

$$3t = 6-2\left(\frac{2}{3}\right) \qquad 1+3t = 3+4\left(\frac{2}{3}\right)$$
$$3t = 6-\frac{4}{3} \qquad 1+3t = 3+\frac{8}{3}$$
$$3t = \frac{18}{3}-\frac{4}{3} \qquad 3t = 2+\frac{8}{3}$$
$$\frac{1}{3}\left(3t = \frac{14}{3}\right)\frac{1}{3} \qquad \frac{6}{3}+\frac{8}{3}$$
$$t = \frac{14}{9} \qquad 3t = \frac{14}{3}$$
$$t = \frac{14}{9}$$

plug s & t into eq ③
s & t have to satisfy all equations

$$1+5\left(\frac{14}{9}\right) \overset{?}{=} 1+7\left(\frac{2}{3}\right)$$
$$1+\frac{70}{9} \overset{?}{=} 1+\frac{14\cdot 3}{3\cdot 3}$$
$$1+\frac{70}{9} \neq 1+\frac{42}{9}$$

$\therefore$ the lines do not intersect

3. *a*) (4 pts) Let $\mathbf{v}$ be a nonzero vector (in $\mathbb{R}^3$), and let $\mathbf{u}$ be any vector (in $\mathbb{R}^3$). By using the dot product, write down a formula for the projection $\mathbf{u}_{\parallel\mathbf{v}}$ of $\mathbf{u}$ along $\mathbf{v}$.

*b*) (6 pts) Recall that we defined the orthogonal component $\mathbf{u}_{\perp\mathbf{v}}$ of $\mathbf{u}$ with respect to $\mathbf{v}$ by

$$\mathbf{u}_{\perp\mathbf{v}} = \mathbf{u} - \mathbf{u}_{\parallel\mathbf{v}}.$$

By using your answer to part *a*), show that $\mathbf{u}_{\perp\mathbf{v}}$ is orthogonal to $\mathbf{v}$.

(a) $\vec{u}_{\|\vec{v}} = \boxed{\left(\frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\right)\vec{v}}$ 4

(b) to prove that two vectors are orthogonal, show that their dot product is equal to 0.

$$\vec{u}_{\perp\vec{v}} \cdot \vec{v} = 0$$

decomposition $\left(\vec{u} - \vec{u}_{\|\vec{v}}\right)\cdot\vec{v} = 0$

distribute $\vec{u}\cdot\vec{v} - \vec{u}_{\|\vec{v}}\cdot\vec{v} = 0$

$$\vec{u}\cdot\vec{v} - \left(\left(\frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\right)\vec{v}\right)\cdot\vec{v} = 0$$

$$\vec{u}\cdot\vec{v} - \left(\left(\frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|\,\|\vec{v}\|}\right)\vec{v}\right)\cdot\vec{v} = 0$$

definition of unit vector $\vec{u}\cdot\vec{v} - \left(\frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|}\left(\cancel{\frac{\vec{v}}{\|\vec{v}\|}}\right)\right)\cdot\vec{v} = 0$ ???

you're subtracting a vector from a scalar...

$$\vec{u}\cdot\vec{v} - \left(\frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|}\right)\cdot\vec{v} = 0$$ ???

scalar multiplication (rearranging) $\vec{u}\cdot\vec{v} - \left(\vec{u}\,\cancel{\frac{\vec{v}}{\|\vec{v}\|}}\right)\cdot\vec{v} = 0$

$$\vec{u}\cdot\vec{v} - \vec{u}\cdot\vec{v} = 0$$

$$0 = 0$$

$\boxed{\therefore \vec{u}_{\perp\vec{v}} \text{ is orthogonal to } \vec{v}}$ 3

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4. (10 pts) Find the area of the triangle with vertices $P = (1, 1, 5)$, $Q = (3, 4, 3)$, and $R = (1, 5, 7)$.

$$A_\Delta = \frac{\|\vec{v} \times \vec{w}\|}{2}$$

$\overrightarrow{PQ} = \langle 2, 3, -2 \rangle$
$\overrightarrow{PR} = \langle 0, 4, 2 \rangle$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & -2 \\ 0 & 4 & 2 \end{vmatrix}$$

$$= \vec{i}(6+8) - \vec{j}(4) + \vec{k}(8)$$

$$= 14\vec{i} - 4\vec{j} + 8\vec{k}$$

$$= \langle 14, -4, 8 \rangle$$

$$\|\overrightarrow{PQ} \times \overrightarrow{PR}\| = \sqrt{14^2 + 16 + 64}$$

$$= \sqrt{196 + 16 + 64}$$

$$= \sqrt{276}$$

$$= 2\sqrt{69}$$

$$A_\Delta = \frac{\|\overrightarrow{PQ} \times \overrightarrow{PR}\|}{2} = \frac{2\sqrt{69}}{2}$$

$$= \boxed{\sqrt{69} \text{ units}^2}$$

$14 \times 14$: 56, 14, 196

16 + 64 = 80

196 + 80 = 276

$276 \div 4 = 69$: $-24$, 36