MATH 32A: CALCULUS OF SEVERAL VARIABLES
SPRING 2017 - LECTURE 3
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MIDTERM 1

Your Name [redacted]

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By signing below, you confirm that you did not cheat on this exam. No exam booklet without a signature will be graded.

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INSTRUCTIONS

- Please do not open this booklet until you are told to do so.
- You are only to use items necessary for writing. No other devices of any kind are permitted.
- No books or notes.
- If you have a question at any time during the exam, please raise your hand.
- You will receive points only for work written on the numbered pages. Please use the reverse side as scratch paper.
- Make sure to write legibly. Illegible work will not be graded.
- Make sure to show all your work and justify your answers fully.
- If you finish early, please wait in your seat until the time is called.

SCORE

1. 10
2. 9
3. 10
4. 10
5. 10

TOTAL 49

1. (10 pts) A force **F** is applied to each of two ropes attached to opposite ends of a 100-kg wagon and each making an angle of 30 degrees with the horizontal. What is the maximum magnitude of **F** that can be applied without lifting the wagon off the ground?

$\vec{F}_g = \langle 0, -100 \times 9.8N \rangle$

$= \langle 0, -980N \rangle$

$\theta_1 = 30°$

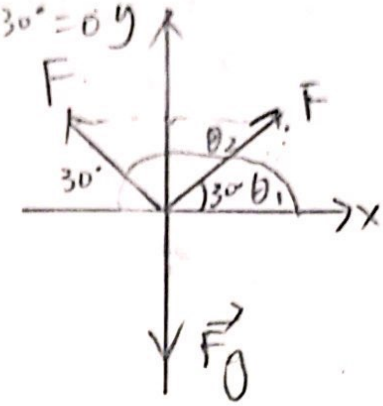
$\theta_2 = 180 - 30° = 150°$

$Proj_y F = \|F\| \sin\theta$

$\vec{F}_{left} = \langle \|\vec{F}\| \cos 150°, \|\vec{F}\| \sin 150° \rangle$

$\vec{F}_{right} = \langle \|\vec{F}\| \cos 30°, \|\vec{F}\| \sin 30° \rangle$

$\|\vec{F}\| \cos 150° + \|\vec{F}\| \cos 30° = 0$



$\|F\| \sin 30° + \|F\| \sin 150 - 980 = 0$

$\frac{1}{2}\|F\| + \frac{1}{2}\|F\| = 980$

$\boxed{\|F\| = 980 \text{ N. max}}$

10

3. (10 pts) Determine whether $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$ define the same line, where

$$\mathbf{r}_1(t) = \langle 3, -1, 5\rangle + t\langle 8, 12, -6\rangle$$

and

$$\mathbf{r}_2(t) = \langle 11, 11, -2\rangle + t\langle 4, 6, -3\rangle.$$

Make sure to justify your answer fully.

1° If $\vec{r_1}(t)$ and $\vec{r_2}(t)$ parallel.

direction vector of $\vec{r_1}(t) \to \vec{v} = \langle 8, 12, -6\rangle$

direction vector of $\vec{r_2}(t) \to \vec{w} = \langle 4, 6, -3\rangle$

$\vec{v} = \langle 8, 12, -6\rangle = 2\langle 4, 6, -3\rangle = 2\vec{w}$

$\vec{v} = 2\vec{w} \Rightarrow \vec{v}$ and $\vec{w}$ parallel $\Rightarrow \vec{r_1}(t)$ and $\vec{r_2}(t)$ parallel.

✓

2° Common point, Let $t=0$ for $\vec{r_1}(t)$

$\vec{r_1}(0) = \langle 3, -1, 5\rangle$.

Check if $\vec{r_2}(t)$ also pass through $\langle 3, -1, 5\rangle$.

for $\vec{r_2}(t)$, $x = 4t+11$, $y = 6t+11$, $z = -3t-2$

$$\begin{cases} 4t+11 = 3 \\ 6t+11 = -1 \\ -3t-2 = 5 \end{cases} \Rightarrow \begin{cases} t = -2 \\ t = -2 \\ t = -\frac{7}{3} \end{cases} \Rightarrow \text{no solution} \Rightarrow \text{don't pass through } \langle 3, -1, 5\rangle$$

Therefore. $\vec{r_1}(t)$ and $\vec{r_2}(t)$ $\boxed{\text{don't not}}$, define the same line, but they are parallel.

✓

2. (10 pts) Find a vector parametrization for all lines in $\mathbf{R}^3$ that pass through the point

$$P = (1,1,1)$$

and are perpendicular to the line

$$\mathbf{r}(t) = \langle 1,1,1\rangle + t\,\langle 1,0,-1\rangle.$$

The direction vector in your parametrization should have only one free variable (apart from t).

Let the direction vector of line perpendicular to $r(t)$, be $\vec{v_1}$.

The direction vector of $r(t)$ be $\vec{v_2} = \langle 1,0,-1\rangle$

Let $\vec{v_1} = \langle a,b,c\rangle$

Then $\vec{v_1}\cdot\vec{v_2} = 0$ | perpendicular.

$\langle a,b,c\rangle\cdot\langle 1,0,-1\rangle = 0$

$a - c = 0$

$a = c$.

The lines perpendicular to the line $r(t)$ with $P = (1,1,1)$ can be parametrized by

$r(t) = \vec{r_0} + t\vec{v_1}$

$= \vec{OP} + t\vec{v_1}$

$= \langle 1,1,1\rangle + t\langle a, b, a\rangle$ | $a = c$.

$= \cancel{\boxed{\langle 1,1,1\rangle + t\langle 1, e, 1\rangle,\ e\in\mathbb{R}}}$

divide by b

$t\langle \frac{a}{b}, 1, \frac{a}{b}\rangle$

$t\langle d, 1, d\rangle$

or $b=0$

or

$t\langle 0,1,0\rangle$

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or $\langle \cos\theta, \sin\theta, \cos\theta\rangle$

4. (10 pts) Show that the four points

$$P = (2,4,4),\ Q = (3,1,6),\ R = (2,8,0),\ S = (\cancel{7}\,6, 3, 1)$$

all lie in a single plane.

1° Find the plane formed by P, Q, R

$$\vec{PQ} = \langle 3-2,\ 1-4,\ 6-4 \rangle = \langle 1, -3, 2 \rangle$$

$$\vec{PR} = \langle 2-2,\ 8-4,\ 0-4 \rangle = \langle 0, 4, -4 \rangle$$

$$\vec{n} = \vec{PQ} \times \vec{PR} = \langle 1, -3, 2 \rangle \times \langle 0, 4, -4 \rangle$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -3 & 2 \\ 0 & 4 & -4 \end{vmatrix} = 4\vec{i} + 4\vec{j} + 4\vec{k} = \langle 4, 4, 4 \rangle$$

$$\mathcal{P}: \vec{n} \cdot \langle x, y, z \rangle = \vec{n} \cdot \vec{OP}$$

$$4x + 4y + 4z = \underbrace{4 \times 2}_{8} + \underbrace{4 \times 4}_{16} + \underbrace{4 \times 4}_{16} = 40$$

$$4x + 4y + 4z = 40$$

$$x + y + z = 10$$

2° Check if S lies in the plane formed by P, Q, R.

$S = (6, 3, 1)$, plug in to $\mathcal{P}: x + y + z = 10$

$6 + 3 + 1 = 10 \Rightarrow$ S is on the $\mathcal{P}$ formed by P, Q, R

$$\Rightarrow \boxed{\begin{array}{l} P, Q, R, S \text{ lie in a single plane} \\ \mathcal{P}: x + y + z = 10 \end{array}}$$

5. (10 pts) Let L denote the line of intersection of the planes

$$x - y - z = 1$$

and

$$2x + 3y + z = 2.$$

Find a vector parametrization for the line L.

1° Find two points both lie in $x-y-z=1$ and $2x+3y+z=2$

let $z=1$

$$\begin{cases} x-y=2 \\ 2x+3y=1 \end{cases} \Rightarrow \begin{cases} x=\frac{7}{5} \\ y=-\frac{3}{5} \end{cases}$$

let $z=2$

$$\begin{cases} x-y=3 \\ 2x+3y=0 \end{cases} \Rightarrow \begin{cases} x=\frac{9}{5} \\ y=-\frac{6}{5} \end{cases}$$

points $\left(\frac{7}{5}, \frac{-3}{5}, 1\right)$ and $\left(\frac{9}{5}, -\frac{6}{5}, 2\right)$ both lie on the two planes above.

2° Find the direction vector $\vec{v}$ of the line L formed by these two points

$$\vec{v} = \left\langle \frac{9}{5} - \frac{7}{5}, -\frac{6}{5} + \frac{3}{5}, 2-1 \right\rangle$$

$$= \left\langle \frac{2}{5}, \frac{-3}{5}, 1 \right\rangle$$

3° The parametrization for the line L is

$$\boxed{L: \vec{r}(t) = \left\langle \frac{7}{5}, -\frac{3}{5}, 1 \right\rangle + t \left\langle \frac{2}{5}, -\frac{3}{5}, 1 \right\rangle}$$