Final (Math 32A)

Tuesday, March 20th

You will have 24 hours to complete this exam. It will be due at 12:00 am on Saturday, 3/21. Please upload your solutions to CCLE before the deadline. You are strongly encouraged to use the solution template posted on CCLE for your solutions. If you do not use the template, make sure you still format your solutions in the same way (so name, ID and *signature* on page 1, Problem 1 on page 2, Problem 2 on page 3, and so on).

The exam is open book/open notes. You can use any materials including the notes, your homeworks and quizzes, and anything you can find on the internet. However you are not allowed to get help from any other person. This includes talking to someone in person, talking to someone online, or asking for help on any sort of online forum. Also it includes asking specific questions about the questions on the final, OR more general questions about the material in the course.

While you may use any resources you can find, your submitted solutions cannot rely on anything other than pencil and paper. That is, even if you used some advanced resources to get your answer originally, or to check that it is correct, in order for your solution to be considered correct, you must include all of the steps you would have taken to get the solution if you did not have access to a calculator or a computer.

Show your work for these problems, don't just give an answer. Unless otherwise stated, you will *not* receive full credit for giving the correct answer with no explanation. You may use any results from class, the textbook or the homework sets, but please make it clear when you are doing so. You may still earn partial credit even if your final answer is incorrect.

Question	Points	Score
1	30	
2	15	
3	20	
4	10	
5	20	
6	20	
7	15	
8	25	
9	25	
10	20	
Total:	200	

Name:______________________________

1. [30 pts] Find the indicated derivatives:

(a) [5 pts] $\dfrac{\partial}{\partial y}\left[ye^{x^2y^2} + \dfrac{1}{\sqrt{x+y}} + \ln(\ln(25+\sin(e^x)))\right]$

(b) [8 pts] $\dfrac{\partial^3}{\partial x\partial y\partial z}\left[e^{xyz} + x^3y^4z^5 + \dfrac{\tan(\ln(y^2+z^4e^y))}{\sqrt{y^7+z^{14}}}\right]$

(c) [8 pts] The directional derivative $D_{\mathbf{u}}f(-1,1,2)$, where $f(x,y,z) = \dfrac{x^2z}{y^3}$ and $\mathbf{u}$ is the unit vector pointing from $(-1,1,2)$ to the origin.

(d) [9 pts] Let $f(x,y) = \sqrt{x^2+y^2}$, and let $g(t) = f\left(4t^3 - t^2,\, 5\ln(t) - 4\cos(t-1)\right)$. Find $g'(1)$.

Name:________________________________

2. [15 pts] Let $\mathcal{C}$ be the curve defined by the vector valued function

$$\mathbf{r}(t) = \left\langle 2e^t + 4, 5 - t, e^{2t} \right\rangle$$

on the interval $-1 \leq t \leq 2$.

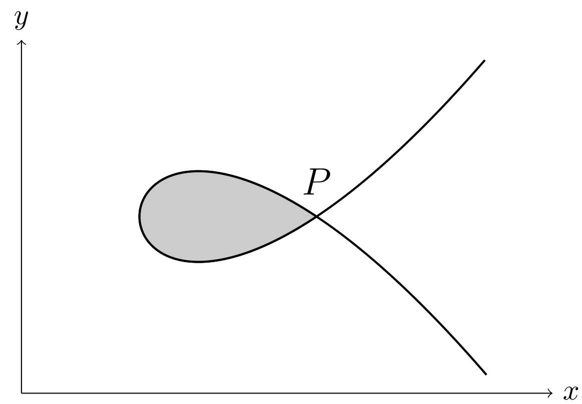
(a) [8 pts] Find the arc length of $\mathcal{C}$ (from the point $\mathbf{r}(-1)$ to the point $\mathbf{r}(2)$).

(b) [7 pts] Find the curvature of $\mathcal{C}$ at the point $(6, 5, 1)$.

Name:________________________________

3. [20 pts] The curve $\mathcal{C}$ in the xy-plane is given by the parametric equations

$$x = 3t^2 + 2 \qquad\qquad y = 2t^3 - 2t + 3.$$



(a) [7 pts] Find the (x, y) coordinates of the point P, where the graph of $\mathcal{C}$ intersects itself.

(b) [13 pts] Find the area of the shaded region, enclosed by the graph of $\mathcal{C}$.

Name:__________________________________

4. [10 pts] $f(x, y)$ is a differentiable function satisfying

$$f(0,0) = 4, \qquad f_x(x,y) = 3x^2 + 8xy^3, \qquad \text{and} \qquad f_y(x,y) = 12x^2y^2 + e^y.$$

Find $f(x, y)$.

Name:____________________________________

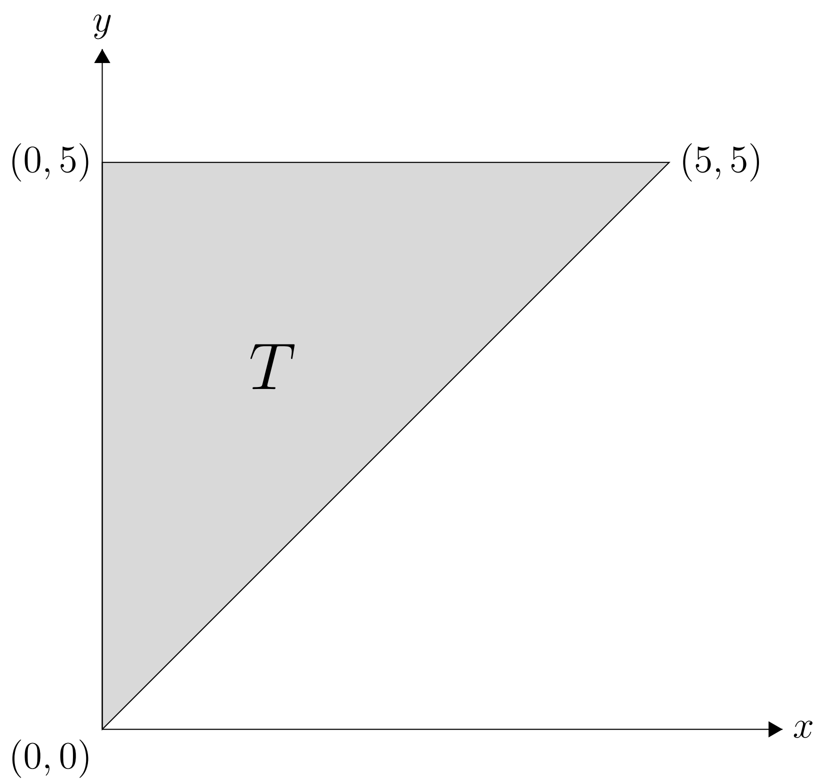
5. [20 pts] Find all critical point(s) of the function

$$f(x,y) = (x^2 + 2xy)e^{-y}.$$

Classify each one as a local maximum, a local minimum, or a saddle point.

Name:________________________________

6. [20 pts] Let $f(x,y) = y^2 - xy + 6x - 2y$. Find the (global) maximum and minimum values which f takes in the triangular region T, with vertices at $(0,0)$, $(0,5)$ and $(5,5)$ (i.e. the region defined by the inequalities $x \geq 0$, and $x \leq y \leq 5$), and the points at which they occur.



Name:________________________________

7. [15 pts] The function $f(x,y)$ is differentiable, but you do not have a formula for it. The following values of f and of its partial derivatives are known:

$f(-2,-2)=5$ $f_x(-2,-2)=4$ $f_y(-2,-2)=3$	$f(-2,-1)=-1$ $f_x(-2,-1)=-1$ $f_y(-2,-1)=-1$	$f(-2,0)=0$ $f_x(-2,0)=4$ $f_y(-2,0)=-5$	$f(-2,1)=10$ $f_x(-2,1)=-2$ $f_y(-2,1)=4$	$f(-2,2)=12$ $f_x(-2,2)=11$ $f_y(-2,2)=-3$
$f(-1,-2)=8$ $f_x(-1,-2)=2$ $f_y(-1,-2)=1$	$f(-1,-1)=11$ $f_x(-1,-1)=13$ $f_y(-1,-1)=17$	$f(-1,0)=-2$ $f_x(-1,0)=-7$ $f_y(-1,0)=3$	$f(-1,1)=6$ $f_x(-1,1)=9$ $f_y(-1,1)=1$	$f(-1,2)=9$ $f_x(-1,2)=0$ $f_y(-1,2)=-1$
$f(0,-2)=-8$ $f_x(0,-2)=-5$ $f_y(0,-2)=10$	$f(0,-1)=11$ $f_x(0,-1)=-6$ $f_y(0,-1)=7$	$f(0,0)=0$ $f_x(0,0)=0$ $f_y(0,0)=0$	$f(0,1)=1$ $f_x(0,1)=6$ $f_y(0,1)=3$	$f(0,2)=3$ $f_x(0,2)=3$ $f_y(0,2)=3$
$f(1,-2)=11$ $f_x(1,-2)=0$ $f_y(1,-2)=0$	$f(1,-1)=0$ $f_x(1,-1)=-3$ $f_y(1,-1)=-7$	$f(1,0)=3$ $f_x(1,0)=-6$ $f_y(1,0)=9$	$f(1,1)=4$ $f_x(1,1)=2$ $f_y(1,1)=3$	$f(1,2)=2$ $f_x(1,2)=-7$ $f_y(1,2)=5$
$f(2,-2)=3$ $f_x(2,-2)=1$ $f_y(2,-2)=8$	$f(2,-1)=6$ $f_x(2,-1)=9$ $f_y(2,-1)=1$	$f(2,0)=6$ $f_x(2,0)=-4$ $f_y(2,0)=-2$	$f(2,1)=-7$ $f_x(2,1)=5$ $f_y(2,1)=8$	$f(2,2)=2$ $f_x(2,2)=2$ $f_y(2,2)=2$

Let $g(t)=f\left(\ln(t)-\sin(1-t),\, t^5-t^3-t\right)$. Find $g'(1)$.

Be clear about which values from the table you are using!

Name:________________________________

8. [25 pts] The function $z = f(x, y)$ is defined implicitly by the equation

$$z^7 + x^2yz^4 + xe^yz = 4.$$

(that is, the point $(x, y, f(x, y))$ is a point on the surface $z^7 + x^2yz^4 + xe^yz = 4$ for every point (x, y) in the domain of f.)

(a) [6 pts] Find $f(3, 0)$. [*Hint: Trying to solve the equation directly for z may be too hard, but if you plug in $x = 3$ and $y = 0$ there should be an "obvious" value of z that works. Show that it's the only value of z that works.*]

(b) [10 pts] Find the gradient $\nabla f(3, 0)$.

(c) [6 pts] Find the *linearization* $L(x, y)$ for $f(x, y)$ at $(3, 0)$ (that is, a linear function $L(x, y)$ with $L(x, y) \approx f(x, y)$ for $(x, y) \approx (3, 0)$).

(d) [3 pts] Use the approximation from part (c) to estimate $f(2.98, 0.01)$.

Name:________________________________

9. [25 pts] Compute the following limits, or show that they do not exist. Explain your reasoning, don't just give an answer with no explanation:

(a) [5 pts] $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{|x| + |y|}$

(b) [10 pts] $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^6 + y^4}$

(c) [10 pts] $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{xy + xz + yz}{x^2 + y^2 + z^2}$

Name:________________________________

10. [20 pts] Let ℓ be the line defined by the vector valued function $\mathbf{r}(t) = \langle 6 - t, t, 0 \rangle$ and let $\mathcal{S}$ be the surface with equation

$$2x^2 + y^2 + z = 18$$

There are two distinct planes $\mathcal{P}_1$ and $\mathcal{P}_2$ that contain the line ℓ and are tangent to the surface $\mathcal{S}$. Find equations (involving just the variables x, y and z and no other variables) for the planes $\mathcal{P}_1$ and $\mathcal{P}_2$, and the points at which they are tangent to the surface $\mathcal{S}$.