Math 32A, Fall 2015, UCLA Instructor: Steven Heilman

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(By signing here, I certify that I have taken this test while refraining from cheating.)
(3A Tu Cutler, 3B Th Cutler, 3C Tu Boozer, 3D Th Boozer, 3E Tu Flapan, 3F Th Flapan)

Mid-Term 2

This exam contains 7 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- You have 50 minutes to complete the exam, starting at the beginning of class.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this. Scratch paper appears at the end of the document.

Problem	Points	Score
1	13	13
2	10	10
3	10	10
4	10	10
5	15	6
Total:	58	49

Do not write in the table to the right. Good luck![a]

1. (a) (3 points) Let $f(x, y) = x^2 + y^2$. Compute the partial derivative f_{xx}.

$f_x = 2x$

$f_{xx} = \boxed{2}$

(b) (5 points) Let $f(u, v, w, x, y, z) = u^2/v + vxyz + \sin(xwv)$. Compute the partial derivative f_{xvz}.

$f_z = vxy$

$f_{vz} = xy$

$f_{xvz} = \boxed{y}$

$$\frac{\frac{1}{1+x}}{1} = \frac{\frac{1}{1+0}}{1} = 1$$

(c) (5 points) Compute the following limit:

$$\lim_{(x,y)\to(0,2)} (1 + x)^{y/x}.$$

Let L be the limit, ln L is log of limit.

$$\lim_{(x,y)\to(0,2)} \ln\left[(1+x)^{\frac{y}{x}}\right]$$

$$= \lim_{(x,y)\to(0,2)} \frac{y}{x} \ln(1+x)$$

$$= \lim_{(x,y)\to(0,2)} y \lim_{(x,y)\to(0,2)} \frac{\ln(1+x)}{x}$$

$$= 2(1) = 2$$

$\ln L = 2, \ l$ ... e^2

$3(2^2)(3^2)$

$\begin{array}{r} 2 \\ 27 \\ 4 \\ \hline 108 \end{array}$ $2(3^3)(2)$ $\begin{array}{r} 2 \\ 27 \\ 4 \\ \hline 108 \end{array}$

2. (10 points) Let $f(x, y) = x^2y^3$. Compute the gradient $\nabla f(x, y)$. Then, find the tangent plane to the surface $z = f(x, y)$ at the point $(a, b) = (2, 3)$.

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$$\boxed{\nabla f(x,y) = (2y^3x, 3y^2x^2)}$$

$\begin{array}{r} 12 \\ 9 \\ \hline 108 \end{array}$

$\nabla f(x,y)$ at $(2,3)$ is $(108, 108)$

$f(a,b) = 2^2(3^3) = 4(27) = 108$

$z = ((x,y)-(2,3)) \cdot (108,108) + 108$

$$\boxed{z = 108(x-2) + 108(y-3) + 108}$$

729 × 2 = 1458

$3\overline{)1458}$ = 486

486 + 486 = 972

486 − 243 = 729

486 + 486 = 972

729 − 243 = 486

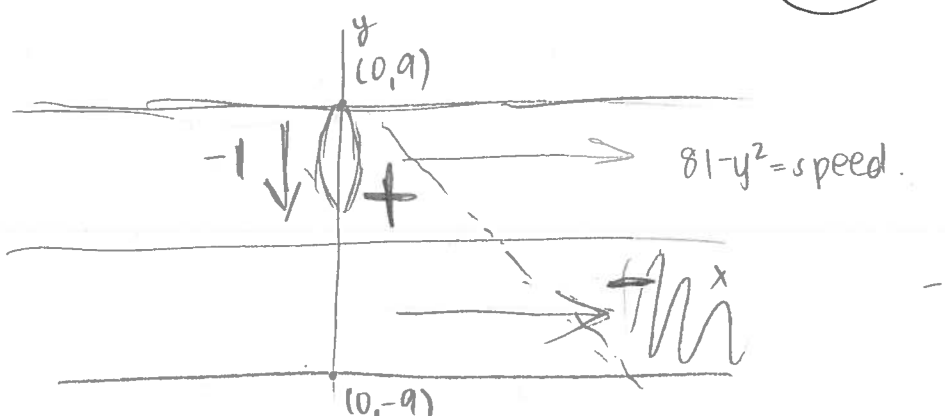
3. (10 points) Suppose your initial position in the plane is $(0, 9)$. Between the lines $y = 9$ and $y = -9$ is a river. The river's speed at the point (x, y) is $81 - y^2$, where the river runs ~~in the direction~~ of the positive x-axis. Suppose you are in a boat which begins with a constant speed of 1, in the negative y-direction. (So, the velocity of the boat in the y-direction will always be -1.) What will be your position when you reach the bottom of the river? That is, what is your position when you reach the line $y = -9$?

$3\overline{)729}$ = 243

$81 \times 9 = 729$

$9 \cdot 9 = 81$, $\times 9 = 729$

$-729 - (-243)$



Displacement = integral of velocity

$$r(y) = \int_0^y r'(y)\,dy \quad \longrightarrow \quad \int_9^{-9} 81 - y^2\,dy$$

$$\Big|_{-9}^{-9} 81y - \frac{y^3}{3}$$ Neg. and pos. will cancel out

$$\Big|_0^9 81y - \frac{y^3}{3} \quad [486]$$

since this is only half, multiply by 2.

$= 486(2) = 972$

$$\boxed{(972, -9)}$$

4. (10 points) Find a function $f(x, y)$ such that

$$\frac{\partial f}{\partial x} = 1 + e^x \cos y,$$

$$\frac{\partial f}{\partial y} = 14y - e^x \sin y,$$

and such that $f(\ln 2, 0) = \ln 2$. (As usual, you must show your work to receive full credit.)

$$\frac{\partial f}{\partial x} = 1 + e^x \cos y \quad \text{constant if } y \text{ is constant.}$$

$$f = x + e^x \cos y + C$$

const. if x is const.

$$\frac{\partial f}{\partial y} = 14y - e^x \sin y$$

$$f = 7y^2 + e^x \cos y + C$$

overall: $f(x,y) = x + e^x \cos y + 7y^2 + C$

$$f(\ln 2, 0) = \ln 2 + \underbrace{e^{\ln 2}}_{2} + 0 + C = \ln 2$$

$$= \ln 2 + 2 + 0 + C = \ln 2$$

$$C = -2$$

$\therefore$ $\boxed{f(x,y) = x + e^x \cos y + 7y^2 - 2}$

5. (15 points) Let D be the solid region in Euclidean space $\mathbf{R}^3$ defined as the set of all (x, y, z) such that $x^2 + y^2 + z^2 \leq 4$, $x^2 + y^2 - 3z^2 \geq 0$ and $x^2 + y^2 \geq 1$. Note that D is a solid region, so its boundary is a surface. Let E denote the boundary surface of D. (If the solid region D were dipped in paint, then the boundary of D is the outer part of D that is covered in paint.)

Let B be the region in Euclidean space $\mathbf{R}^3$ defined as the set of all (x, y, z) such that $y = x$, $x \geq 0$ and $y \geq 0$. Then E and B are surfaces.

Parametrize the intersection of E and B. (Make sure to parametrize the entire intersection. You **MUST** specify the domain of your parameter for any parametrization you give.) (Any parametrization that you write **MUST** use z as a parameter. That is, any parametrization you write must be of the form $r(z) = (x(z), y(z), z)$, where x and y are both functions of z.)

$x^2 = 1 - y^2$

$x^2 + y^2 - 3z^2 \geq 0$

$1 - \cancel{y^2} + \cancel{y^2} - 3z^2 \geq 0$

$1 - 3z^2 \geq 0$

$1 \geq 3z^2$

$z^2 \leq \frac{1}{3}$

$x^2 + y^2 + z^2 \leq 4$

$1 \leq x^2 + y^2 \leq 3\frac{2}{3}$, $z^2 \leq \frac{1}{3}$

$\frac{9}{3} + \frac{2}{3} = \frac{11}{3}$

$y = x$

N = 2

$x^2 + x^2 + z^2 \leq 4$

$2x^2 \leq 4 - z^2$

$x^2 \leq \frac{4 - z^2}{2}$

$-2 \leq z \leq 2$

$x^2 + x^2 - 3z^2 \geq 0$

$2x^2 \geq 3z^2$

$x^2 \geq \frac{3}{2} z^2$

$x \geq \sqrt{\frac{3}{2} z^2}$

$\left(\sqrt{\frac{4 - z^2}{2}}, \sqrt{\frac{4 - z^2}{2}}, z \right)$ $\quad -\frac{1}{9} \leq \cancel{z} \leq \frac{1}{9}$

$\left(\sqrt{\frac{3}{2} z^2}, \sqrt{\frac{3}{2} z^2}, z \right)$ $\quad -\frac{1}{9} \leq \cancel{z} \leq \frac{1}{9}$

(Scratch paper)