1. (a) (3 points) Let $f(x, y) = x^2 + y^2$. Compute the partial derivative f_{xx}.

$f_x = 2x + 0$

$f_{xx} = \boxed{2}$

(b) (5 points) Let $f(u, v, w, x, y, z) = u^2/v + vxyz + \sin(xwv)$. Compute the partial derivative f_{xvz}.

f_{xvz} : $f_z = 0 + vxy + 0$

$\leftarrow$

$f_{vz} = xy$

$f_{xvz} = \boxed{y}$

(c) (5 points) Compute the following limit:

$$\lim_{(x,y)\to(0,2)} (1 + x)^{y/x}.$$

$\lim_{(x,y)\to(0,2)} (1+x)^{y/x}$; $(1+x)^{y/x} = L$ $\qquad \ln(L) = \ln(1+x)^{y/x}$

$\ln(L) = \dfrac{y \ln(1+x)}{x}$

$\ln\left(\lim_{(x,y)\to(0,2)} \dfrac{y \ln x}{x}\right) = \lim_{(x,y)\to(0,2)} y \cdot \lim_{(x,y)\to(0,2)} \dfrac{\ln(1+x)}{x}$

$= 2 \cdot \lim_{(x,y)\to(0,2)} \dfrac{\frac{1}{1+x}}{1}$

$= 2 \cdot 1 = 2$

$e^\wedge \left(\lim_{(x,y)\to(0,2)} \ln(L)\right) = e^\wedge(2)$

$\lim_{(x,y)\to(0,2)} (1+x)^{y/x} = \underbrace{e^2}$

$2\cdot4$
$\times 27$
$\cancel{3}\ 8$

2. (10 points) Let $f(x, y) = x^2y^3$. Compute the gradient $\nabla f(x, y)$. Then, find the tangent plane to the surface $z = f(x, y)$ at the point $(a, b) = (2, 3)$.

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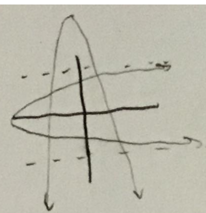
$$\nabla f(x,y) = (f_x, f_y)$$
$$= \underline{(2xy^3, 3x^2y^2)}$$

$$L(x,y) = f(2,3) + \nabla f(2,3) \cdot ((x,y) - (2,3))$$
$$= 2^2 \cdot 3^3 + (2(2)(3^3), 3 \cdot 2^2 \cdot 3^2) \cdot (x-2, y-3)$$
$$= 108 + (108, 108) \cdot (x-2, y-3)$$
$$= 108 + 108x - 2(108) + 108y - 3(108)$$
$$z = 108(x + y - 4)$$

$x = 81 - y^2$
$y^2 = 81 - x$
$y = \sqrt{81 - x}$



3. (10 points) Suppose your initial position in the plane is $(0, 9)$. Between the lines $y = 9$ and $y = -9$ is a river. The river's speed at the point (x, y) is $81 - y^2$, where the river runs in the direction of the positive x-axis. Suppose you are in a boat which begins with a constant speed of 1, in the negative y-direction. (So, the velocity of the boat in the y-direction will always be -1.) What will be your position when you reach the bottom of the river? That is, what is your position when you reach the line $y = -9$?

$$r_y(t) = r(0) + tv + at^2$$
$$= 9 + (-t) + 0$$
$$= 9 - t$$

+1 +1

$-9 = 9 - t$, $t = 18$ when $y = -9$ i.e. we're at the bottom

find unit speed parametrization for x direction

$$\text{Speed} = \|v(t)\| = 81 - y^2$$

$$\therefore \quad \sigma(t) = \int_{-9}^{9} 81 - y^2 \, dy$$
$$= 81y - \tfrac{1}{3}y^3 \Big|_{-9}^{9}$$
$$= 81(9) - \tfrac{1}{3}(9^3) - (81(-9) - \tfrac{1}{3}(-9)^3)$$
$$= 2\left(9^3 - \frac{9^3}{3}\right)$$

10

4. (10 points) Find a function $f(x, y)$ such that

$$\frac{\partial f}{\partial x} = 1 + e^x \cos y,$$

$$\frac{\partial f}{\partial y} = 14y - e^x \sin y,$$

and such that $f(\ln 2, 0) = \ln 2$. (As usual, you must show your work to receive full credit.)

-10

$$\int \frac{df}{dx}\,dx = \int 1 + e^x \cos y = \int 1 + \left(\int e^x\,dx \cdot \int \cos y\,dx\right)$$
$$= x + \left(\frac{e^x}{1} \cdot x\cos y\right) + C_x$$

$$\int \frac{df}{dy}\,dy = \int 14y - e^x \sin y = \int 14y\,dy - \left(\int e^x\,dy \cdot \int \sin y\,dy\right)$$
$$= 7y^2 - \left(ye^x \cdot -\cos y\right) + C_y$$

$$f(x,y) = \left(x + e^x x\cos y + C_x,\ 7y^2 + \cos y \cdot ye^x + C_y\right)$$

$$\ln 2 + e^{\ln 2} \cdot \ln 2 \cos 0 + C_x = \ln 2$$

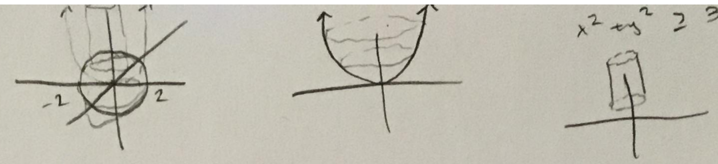
$$\ln 2 + 2(\ln 2)(1) + C_x = \ln 2$$

$$C_x = \ln 2 - 3\ln 2 = -2\ln 2$$

$$7(0)^2 + \cos 0 \cdot (0)(e^{\ln 2}) + C_y = 0$$

$$C_y = 0$$

$$\therefore\ f(x,y) = \left(x + x\cos y e^x - 2\ln 2,\ 7y^2 + y \cdot \cos(y) e^x\right)$$



5. (15 points) Let D be the solid region in Euclidean space $\mathbf{R}^3$ defined as the set of all (x, y, z) such that $x^2 + y^2 + z^2 \le 4$, $x^2 + y^2 - 3z^2 \ge 0$ and $x^2 + y^2 \ge 1$. Note that D is a solid region, so its boundary is a surface. Let E denote the boundary surface of D. (If the solid region D were dipped in paint, then the boundary of D is the outer part of D that is covered in paint.)

Let B be the region in Euclidean space $\mathbf{R}^3$ defined as the set of all (x, y, z) such that $y = x$, $x \ge 0$ and $y \ge 0$. Then E and B are surfaces.

Parametrize the intersection of E and B. (Make sure to parametrize the entire intersection. You **MUST** specify the domain of your parameter for any parametrization you give.) (Any parametrization that you write **MUST** use z as a parameter. That is, any parametrization you write must be of the form $r(z) = (x(z), y(z), z)$, where x and y are both functions of z.)

$z^2 \le 4 - x^2 - y^2$, $z^2 \le \frac{x^2}{3} + \frac{y^2}{3}$, $\frac{x^2}{3} + \frac{y^2}{3} \ge \frac{1}{3}$

$y = x$, $x \ge 0$, $y \ge 0$

$\therefore$ $2x^2 + z^2 \le 4$, $2x^2 - 3z^2 \ge 0$, $2x^2 \ge 1$

$z^2 \le 4 - 2x^2$, $z^2 \le \frac{2}{3}x^2$, $x^2 \ge 1/2$

or $x^2 \le \frac{4-z^2}{2}$, $x^2 \ge \frac{3}{2}z^2$

$x^2 + y^2 \ge 3z^2$

≥ 1

$x^2 \ge 1 - y^2$

$\cos t, \sin t,$

$x^2+y^2+z^2 = 4$, $x^2+y^2 = 3z^2$, $x^2+y^2 = 1$

$2x^2 + z^2 = 4$, $2x^2 = 3z^2$, $2x^2 = 1$

$x^2 = \frac{4-z^2}{2}$, $x^2 = \frac{3z^2}{2}$, $x^2 = 1/2$

$x = \sqrt{\frac{4-z^2}{2}}$, $x = \sqrt{\frac{3z^2}{2}}$, $x = 1/2$

none negative because $x \ge 0$

$$r(z) = \begin{cases} \left(\sqrt{\frac{4-z^2}{2}}, \sqrt{\frac{4-z^2}{2}}, z\right) & \text{for } 0 \le \cancel{z} \le 2 \\ \left(\sqrt{\frac{3z^2}{2}}, \sqrt{\frac{3z^2}{2}}, z\right) & \text{for } z \cancel{\ge} 0 \end{cases}$$

N = 2