Math 32A, Fall 2015, UCLA Instructor: Steven Heilman

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Signature: ____________. Section 3C

(By signing here, I certify that I have taken this test while refraining from cheating.)
(3A Tu Cutler, 3B Th Cutler, 3C Tu Boozer, 3D Th Boozer, 3E Tu Flapan, 3F Th Flapan)

Mid-Term 2

This exam contains 7 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- You have 50 minutes to complete the exam, starting at the beginning of class.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this. Scratch paper appears at the end of the document.

Problem	Points	Score
1	13	12
2	10	9
3	10	10
4	10	10
5	15	8
Total:	58	49

Do not write in the table to the right. Good luck![a]

1. (a) (3 points) Let $f(x, y) = x^2 + y^2$. Compute the partial derivative f_{xx}.

3

$f_x = 2x$

$f_{xx} = 2$

✓

(b) (5 points) Let $f(u, v, w, x, y, z) = u^2/v + vxyz + \sin(xwv)$. Compute the partial derivative f_{xvz}.

5

$f_z = vxy$

$f_{zv} = xy$

$f_{zvx} = y$

$f_{zvx} = \boxed{f_{xvz} = y}$ ✓

because they both exist and are cont

(c) (5 points) Compute the following limit:

4

$$\lim_{(x,y)\to(0,2)} (1 + x)^{y/x}.$$

not justified −1

$$\lim_{(x,y)\to(0,2)} (1+x)^{\frac{y}{x}} = \lim_{y\to 2}\left(\lim_{x\to 0}(1+x)^{\frac{1}{x}}\right)^{y}$$

(by definition of e)

$$= \lim_{y\to 2} e^y = \boxed{e^2}$$

2. (10 points) Let $f(x,y) = x^2y^3$. Compute the gradient $\nabla f(x,y)$. Then, find the tangent plane to the surface $z = f(x,y)$ at the point $(a,b) = (2,3)$.

9

$$\nabla f(x,y) = (2xy^3, 3x^2y^2)$$

$$z = f(a,b) + \nabla f(a,b) \cdot ((x,y) - (a,b))$$

$$= 2^2 \cdot 3^3 + (2\cdot 2\cdot 3^3, 3\cdot 2^2 \cdot 3^2) \cdot ((x-2),(y-3))$$

$$= 108 + (108, 108) \cdot ((x-2),(y-3))$$

$$= 108 + 108x - 2\cdot 108 + 108y - 3\cdot 108$$

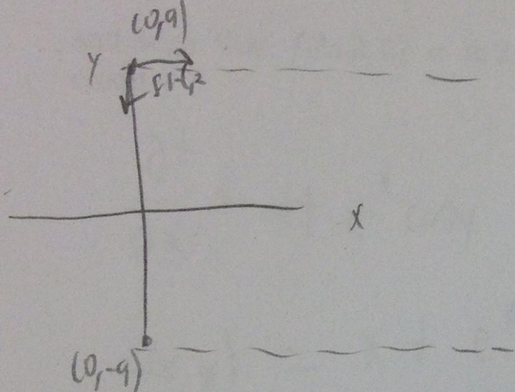
$$= 108x + 108y + 4\cdot 108$$

$$z = 108x + 108y + 432$$

$-$

-1

3. (10 points) Suppose your initial position in the plane is $(0, 9)$. Between the lines $y = 9$ and $y = -9$ is a river. The river's speed at the point (x, y) is $81 - y^2$, where the river runs in the direction of the positive x-axis. Suppose you are in a boat which begins with a constant speed of 1, in the negative y-direction. (So, the velocity of the boat in the y-direction will always be -1.) What will be your position when you reach the bottom of the river? That is, what is your position when you reach the line $y = -9$?



$$v(x,y) = (81-y^2, -1)$$

$$V_{x\text{-direction}} = 81-y^2$$

we will calculate distance to 0 and then double it because velocity is symmetric about the x-axis

distance in x from $y=9$ to 0 $= \left| \int_0^9 81-y^2 \, dy \right|$ — (this should be $\int_9^0$ but it doesn't matter bc absolute value)

$$= \left| \left[81y - \tfrac{1}{3}y^3 \right]_0^9 \right|$$

$$= \left| 81 \cdot 9 - \tfrac{1}{3} \cdot 9^3 - 0 \right|$$

$$= \left| 81 \cdot (9-3) \right|$$

$$= |81 \cdot 6| = 486$$

Therefore, total distance in x-direction is $486 \times 2 = 972$

so it will be at $\boxed{(972, -9)}$

4. (10 points) Find a function $f(x, y)$ such that

$$\frac{\partial f}{\partial x} = 1 + e^x \cos y,$$

$$\frac{\partial f}{\partial y} = 14y - e^x \sin y,$$

and such that $f(\ln 2, 0) = \ln 2$. (As usual, you must show your work to receive full credit.)

$$\frac{\partial f}{\partial x} = 1 + e^x \cos y$$

$$f(x,y) = x + e^x \cos y + C_1$$

$$\frac{\partial f}{\partial y} = 14y - e^x \sin y$$

$$f(x,y) = 7y^2 + e^x \cos y + C_2$$

Therefore $f(x,y) = x + e^x \cos y + 7y^2 + C_3$

so $\ln 2 = f(\ln 2, 0) = \ln 2 + e^{(\ln 2)} \cos 0 + 7 \cdot 0^2 + C_3$

$$\ln 2 = \ln 2 + 2 + C_3$$

$$C_3 = -2$$

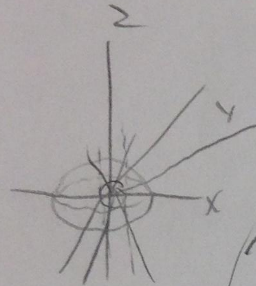
Therefore $\boxed{f(x,y) = x + e^x \cos y + 7y^2 - 2}$

5. (15 points) Let D be the solid region in Euclidean space $\mathbf{R}^3$ defined as the set of all (x, y, z) such that $x^2 + y^2 + z^2 \leq 4$, $x^2 + y^2 - 3z^2 \geq 0$ and $x^2 + y^2 \geq 1$. Note that D is a solid region, so its boundary is a surface. Let E denote the boundary surface of D. (If the solid region D were dipped in paint, then the boundary of D is the outer part of D that is covered in paint.)

Let B be the region in Euclidean space $\mathbf{R}^3$ defined as the set of all (x, y, z) such that $y = x$, $x \geq 0$ and $y \geq 0$. Then E and B are surfaces.

Parametrize the intersection of E and B. (Make sure to parametrize the entire intersection. You **MUST** specify the domain of your parameter for any parametrization you give.) (Any parametrization that you write **MUST** use z as a parameter. That is, any parametrization you write must be of the form $r(z) = (x(z), y(z), z)$, where x and y are both functions of z.)

$x^2+y^3 \geq 3z^2$



edge of cylinder ~~and~~ w/ cone

when $x^2+y^2 \geq 1$ and $x=y$

to when it runs into $x^2+y^2-3z^2 \geq 0$

$2x^2 \geq 1$

$x^2 \geq \frac{1}{2}$

$x \geq \frac{\sqrt{2}}{2}$

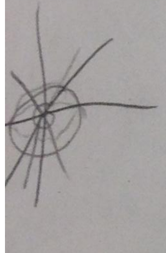
$\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 - 3z^2 \geq 0$

$\frac{1}{2} + \frac{1}{2} - 3z^2 \geq 0$

$3z^2 \leq 1$

$z^2 \leq \frac{1}{3}$

$-\frac{\sqrt{3}}{3} \leq z \leq \frac{\sqrt{3}}{3}$



(all +)

edge of cone until sphere
start from $z = \pm\frac{\sqrt{3}}{3}$ until hits sphere

$z \in [\frac{\sqrt{3}}{3}, 1]$

N=2

edge of sphere ~~and~~ w/ cone and $x=y$

$x^2+y^2+z^2 \leq 4$ from when $x^2+y^2-3z^2 \geq 0$

$z^2 \leq 4-2x^2$

$z^2 = 4-2x^2$

$2x^2 = 4-z^2$

$x^2 = 2-\frac{z^2}{2}$

$x = \pm\sqrt{2-\frac{z^2}{2}}$

$2x^2 \geq 3z^2$

$4-2x^2 \geq z^2$

$4 \geq 4z^2$

$z^2 \leq 1$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, z\right)$ $z \in \left[-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right]$

$\left(\sqrt{2-\frac{z^2}{2}}, \sqrt{2-\frac{z^2}{2}}, z\right)$, $z \in [\frac{\sqrt{3}}{3}, 1]$

$\cancel{\left(-\sqrt{2-\frac{z^2}{2}}, -\sqrt{2-\frac{z^2}{2}}, z\right), z}$