Math 32A, Fall 2015, UCLA Instructor: Steven Heilman

Name: Ramya Sarish UCLA ID: [redacted] Date: 10/19/15

Signature: [redacted].
(By signing here, I certify that I have taken this test while refraining from cheating.)

3A

Mid-Term 1

This exam contains 7 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- You have 50 minutes to complete the exam, starting at the beginning of class.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this. Scratch paper appears at the end of the document.

Do not write in the table to the right. Good luck![a]

Problem	Points	Score
1	13	13
2	10	10
3	12	12
4	10	10
5	10	10
Total:	55	55

Some Formulas:

$u = (x_1, y_1, z_1)$, $w = (x_2, y_2, z_2)$.

$u \cdot w = x_1x_2 + y_1y_2 + z_1z_2$.

$u \times w =$

$$\det \begin{pmatrix} (1,0,0) & (0,1,0) & (0,0,1) \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{pmatrix}.$$

1. (a) (3 points) Find the unit vector which points in the same direction as $(1, 2, 3)$.

$e_v = \frac{(a,b,c)}{\|(a,b,c)\|}$

$$e_v = \frac{(1,2,3)}{\sqrt{1^2+2^2+3^2}}$$

$1+4+9$

$$= \boxed{\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)}$$

(b) (5 points) Let v, w be vectors in $\mathbf{R}^3$ such that $||v|| = 2$, $||w|| = 3$ and such that $v \cdot w = 0$. Find $||v + w||$.

$$\|v+w\|^2 = (v+w)\cdot(v+w)$$

$$= v\cdot v + 2v\cdot w + w\cdot w$$

$$= v\cdot v + w\cdot w$$

$$= \|v\|^2 + \|w\|^2$$

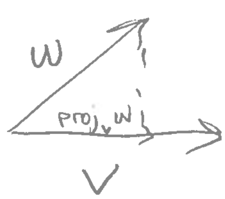
$$= 2^2 + 3^2$$

$$\|v+w\|^2 = 13$$

$$\|v+w\| = \boxed{\sqrt{13}}$$

(c) (5 points) Let $v = (1, 2, 0)$ and let $w = (2, 1, 3)$. Write w as the sum of two vectors r, s, so that $w = r + s$, and such that r is parallel to v, and s is perpendicular to v.

$$w = \text{proj}_v w + (w - \text{proj}_v w)$$



$$\text{proj}_v w = \frac{w\cdot v}{v \cdot v} v$$

$$\text{proj}_v w = \frac{(1,2,0)\cdot(2,1,3)}{(1,2,0)\cdot(1,2,0)} v$$

$$= \frac{2+2}{5} = \frac{4}{5} v = \left(\frac{4}{5}, \frac{8}{5}, 0\right)$$

$w - \text{proj}_v w = (2,1,3) - \left(\frac{4}{5}, \frac{8}{5}, 0\right)$

$= \left(\frac{6}{5}, -\frac{3}{5}, 3\right)$

$w = r + s$

$$\boxed{w = \left(\frac{4}{5}, \frac{8}{5}, 0\right) + \left(\frac{6}{5}, -\frac{3}{5}, 3\right)}$$

100 010 001
1 2 1
1 −1 −2

2. (10 points) Find the angle between the planes $x + 2y + z = 1$ and $x - y - 2z = 0$. Then, find a parametrization for the line of intersection of these planes.

(1,2,1) (1,−1,−2)

$-4-(-1)$

a) $\cos\theta = \left(\frac{(1,2,1)}{\sqrt{1^2+2^2+1^2}} \cdot \frac{(1,-1,-2)}{\sqrt{1^2+(-1)^2+(-2)^2}} \right)$

$= \frac{1-2-2}{6}$

$= \frac{-3}{6} = -\frac{1}{2}$

$\sqrt{3}$ 2 $\pi/3$ −1

$\theta = \cos^{-1}\left(-\frac{1}{2}\right) = \boxed{\frac{2\pi}{3}}$

b) $(1,2,1)\times(1,-1,-2)$

100 010 001
2
−1 −2

$(-4-(-1), (-2-1)(-1), -1-2)$

$(-3, 3, -3)$

$x = y + 2z$, $x + 2y + z = 1$

$y + 2z + 2y + z = 1$

$3y + 3z = 1$, let $y=0$ and $z=\frac{1}{3}$

$\left(\frac{2}{3}, 0, \frac{1}{3}\right)$ is on the line.

Final answer

$-\infty < t < \infty$

$\boxed{(-3,3,-3)t + \left(\frac{2}{3}, 0, \frac{1}{3}\right) = s(t)}$

3. In each of the following items, an equation is listed. This equation describes a quadric surface in $\mathbf{R}^3$. That is, each equation describes a set of points (x, y, z) in three-dimensional Euclidean space. For each such equation, identify the type of surface to which the equation corresponds. You can choose from the following labels: Paraboloid, Hyperboloid, Cone, Ellipsoid, Cylinder. You do not need to show any work in this section. No partial credit will be given in this section.

(a) (2 points) $2x^2 + 3y^2 = 1 - z^2$

PARABOLOID HYPERBOLOID CONE ELLIPSOID CYLINDER
(circle one)

$2x^2+3y^2+z^2=1$

(b) (2 points) $z^2 - 2y^2 + 10z^2 = 0$

~~PARABOLOID~~ ~~HYPERBOLOID~~ ~~CONE~~ ~~ELLIPSOID~~ CYLINDER
(circle one)

$11z^2-2y^2=0$

(c) (2 points) $2x^2 + 3y^2 = z^2 - 1$

PARABOLOID HYPERBOLOID CONE ELLIPSOID CYLINDER
(circle one)

(d) (2 points) $z^2 + 100y^2 = 100$

PARABOLOID HYPERBOLOID CONE ELLIPSOID CYLINDER
(circle one)

(e) (2 points) $y^2 + 4z^2 = x^2 + 1$

PARABOLOID HYPERBOLOID CONE ELLIPSOID CYLINDER
(circle one)

(f) (2 points) $4x = y^2 + z^2/4$

PARABOLOID HYPERBOLOID CONE ELLIPSOID CYLINDER
(circle one)

4. (10 points) Consider the following three points in $\mathbf{R}^5$:

$$(5,3,2,3,1), \quad (5,3,0,1,2), \quad (5,3,0,1,0).$$

Consider the triangle which has these three points as its vertices. Find the area of this triangle.

Parallelogram defined by vectors u and v

$u = (5,3,2,3,1) - (5,3,0,1,0)$

$= (0,0,2,2,1)$ ✓

$v = (5,3,0,1,2) - (5,3,0,1,0)$

$= (0,0,0,0,2)$ ✓

Triangle Area $= \frac{1}{2} \, |u \times v| \longrightarrow$

$$\begin{array}{ccc} 100 & 010 & 001 \\ 2 & 2 & 1 \\ 0 & 0 & 2 \end{array}$$

$= \frac{1}{2} \left(|(4,-4,0)| \right)$ ✓

$= \frac{1}{2} \sqrt{16+16}$

$= \frac{1}{2} \sqrt{32}$

$= \boxed{2\sqrt{2}}$ ✓

(0,0,0)

5. (10 points) Compute: (-2,2,-2) (11,-4,-3)

$$\left[(1,2,1)\times\left(\underbrace{[(1,2,1)\times(3,4,1)]}_{\text{orthogonal to }(1,2,1)}\times\underbrace{[(1,2,1)\times(5,7,9)]}_{\text{orthogonal to }(1,2,1)}\right)\right]\times(4,10,3).$$

Hint: There is a way to do this problem without doing any computations.

Make sure to justify your answer. (If you compute all of these cross products explicitly, and you get the wrong answer, you will get at most half credit.)

STEP 1

(1,2,1)x(3,4,1)

100	010	001
1	2	1
3	4	1

(2-4

(-2,2,-2)

STEP 2

(1,2,1)x(5,7,9)

100	010	001
1	2	1
5	7	9

(11,-4,-3)

STEP 3

(-2,2,-2)x(11,-4,-3)

100	010	001
-2	2	-2
11	-4	-3

(-16-(8), 6+22, 8-22

(-12, -28, -14)

(1,2,1) x (-12,-28,-14)

100	010	001
1	2	1
-12	-28	-14

-14-(-14)

-28-(-28

-14-(-12)

-28-(-24)

-4

(0, 0, 0)

~~(0,0,0) x (4,10,3)~~

~~(0,2,-4) x (4,10,3)~~

~~100~~	~~010~~	~~001~~
~~0~~	~~2~~	~~-4~~
~~4~~	~~10~~	~~3~~

~~0-(-16)~~

~~16~~

~~(20,-16,-8)~~

FINAL ANSWER: $\boxed{(0,0,0)}$

since the vector is crossed with the cross product of 2 result vectors the (0,0,0)

(Scratch paper)