1. (a) (3 points) Find the unit vector which points in the same direction as $(1, 2, 3)$.

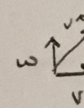
$v = (1,2,3)$

$$e_v = \frac{v}{\|v\|} = \frac{(1,2,3)}{\sqrt{1^2+2^2+3^2}} = \frac{(1,2,3)}{\sqrt{1+4+9}}$$

$$= \frac{(1,2,3)}{\sqrt{14}} = \left(1/\sqrt{14},\ 2/\sqrt{14},\ 3/\sqrt{14}\right)$$

(b) (5 points) Let v, w be vectors in $\mathbf{R}^3$ such that $||v|| = 2$, $||w|| = 3$ and such that $v \cdot w = 0$. Find $||v + w||$.

$$\frac{v\cdot w}{\|v\|\|w\|} = \cos\theta = \frac{0}{2\cdot 3} = 0, \quad \theta = 90^\circ$$



w (for example)

$$\|v+w\| = \sqrt{2^2+3^2} = \sqrt{4+9} = \sqrt{13}$$

(c) (5 points) Let $v = (1, 2, 0)$ and let $w = (2, 1, 3)$. Write w as the sum of two vectors r, s, so that $w = r + s$, and such that r is parallel to v, and s is perpendicular to v.

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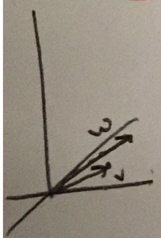

$w = r+s = \text{proj}_v w + n$

$$\text{proj}_v(w) = \left(\frac{w\cdot v}{v\cdot v}\right)(v) = \frac{(2,1,3)\cdot(1,2,0)}{(1,2,0)\cdot(1,2,0)} \cdot (1,2,0)$$

$$= \left(\frac{2+2+0}{1+4+0}\right)\cdot(1,2,0) = \frac{4}{4}\cdot(1,2,0) = (1,2,0)$$

Not quite

$\therefore w = \text{proj}_v w + n$

$\rightarrow (2,1,3) = (1,2,0) + n, \quad n = (1,-1,-3)$

$w = r+s = (1,2,0) + (1,-1,-3)$

2. (10 points) Find the angle between the planes $x + 2y + z = 1$ and $x - y - 2z = 0$. Then, find a parametrization for the line of intersection of these planes.

angle between the planes = angle between their unit normal vectors

P_1: $(1,2,1) \cdot (x,y,z) = 1$ $\qquad$ P_2: $(1,-1,-2) \cdot (x,y,z) = 0$

$n_1 = (1,2,1)$ $\qquad$ $n_2 = (1,-1,-2)$

$$\cos\theta = \frac{n_1 \cdot n_2}{\|n_1\| \|n_2\|} = \frac{(1,2,1) \cdot (1,-1,-2)}{\sqrt{1^2+2^2+1^2} \cdot \sqrt{1^2+(-1)^2+(-2)^2}} = \frac{1-2-2}{\sqrt{36}} = \frac{-3}{6}$$

$$= -1/2$$

$$\therefore \theta = \cos^{-1}(-1/2) = \boxed{2\pi/3}$$

find vector perpendicular to both by finding one $\perp$ to both normal vectors

$$n_1 \times n_2 = \begin{vmatrix} (1,0,0) & (0,1,0) & (0,0,1) \\ 1 & 2 & 1 \\ 1 & -1 & -2 \end{vmatrix}$$

$$= (1,0,0)((2)(-2) - (-1)(1)) - (0,1,0)((1)(-2) - (1)(1)) + (0,0,1)((1)(-1) - (1)(2))$$

$$= (-4+1, -(-2-1), -1-2)$$

$$= (-3, 3, -3)$$

$$\therefore s(t) = t(-3,3,-3) + (x,y,z)$$

~~plug in a point: P_1: $(0,0,1)$ satisfies $0 + 2(0) + 1 = 1$~~ ✓

so $s(t) = \cancel{(0,0,1)} + t(-3,3,-3)$

-1

not on both planes

$x^2 + y^2 = z^2 + 1$ hyp one

z^2 cone

$z^2 - 1$ hyp two

$= z$ ellip. pgra

$= -z$ hyppe pgra

$= c$ cylinder

3. In each of the following items, an equation is listed. This equation describes a quadric surface in $\mathbf{R}^3$. That is, each equation describes a set of points (x, y, z) in three-dimensional Euclidean space. For each such equation, identify the type of surface to which the equation corresponds. You can choose from the following labels: Paraboloid, Hyperboloid, Cone, Ellipsoid, Cylinder. You do not need to show any work in this section. No partial credit will be given in this section.

(a) (2 points) $2x^2 + 3y^2 = 1 - z^2$ $\quad 2x^2 + 3y^2 + z^2 = 1$

PARABOLOID HYPERBOLOID CONE ELLIPSOID CYLINDER
(circle one)

(b) (2 points) $z^2 - 2y^2 + 10z^2 = 0$ $\quad 11z^2 = 2y^2$

PARABOLOID HYPERBOLOID CONE ELLIPSOID CYLINDER
(circle one)

(c) (2 points) $2x^2 + 3y^2 = z^2 - 1$

PARABOLOID HYPERBOLOID CONE ELLIPSOID CYLINDER
(circle one)

(d) (2 points) $z^2 + 100y^2 = 100$

PARABOLOID HYPERBOLOID CONE ELLIPSOID CYLINDER
(circle one)

(e) (2 points) $y^2 + 4z^2 = x^2 + 1$

PARABOLOID HYPERBOLOID CONE ELLIPSOID CYLINDER
(circle one)

$x^2 + y^2 = z$

(f) (2 points) $4x = y^2 + z^2/4$

PARABOLOID HYPERBOLOID CONE ELLIPSOID CYLINDER
(circle one)

4. (10 points) Consider the following three points in $\mathbf{R}^5$:

$$\underset{A}{(5,3,2,3,1)}, \quad \underset{B}{(5,3,0,1,2)}, \quad \underset{C}{(5,3,0,1,0)}.$$ all on the same xy

Consider the triangle which has these three points as its vertices. Find the area of this triangle.

$\dfrac{\left\| \det\begin{pmatrix} \overrightarrow{AB} \\ \overrightarrow{BC} \end{pmatrix} \right\|}{2}$, where $\overrightarrow{AB} = (5,3,0,1,2) - (5,3,2,3,1) = (0,0,-2,-2,1)$

$\overrightarrow{BC} = (5,3,0,1,0) - (5,3,0,1,2) = (0,0,0,0,-2)$

so in $\mathbb{R}^3$ $\overrightarrow{AB} = (-2,-2,1)$

$\overrightarrow{BC} = (0,0,-2)$

$$\det\begin{pmatrix} \overrightarrow{AB} \\ \overrightarrow{BC} \end{pmatrix} = \begin{vmatrix} (1,0,0) & (0,1,0) & (0,0,1) \\ -2 & -2 & 1 \\ 0 & 0 & -2 \end{vmatrix}$$

$= (1,0,0)(4-0) - (0,1,0)(4-0) + (0,0,1)(0-0)$

$= (4,-4,0)$

$\left\| \dfrac{\det\begin{pmatrix} \overrightarrow{AB} \\ \overrightarrow{BC} \end{pmatrix}}{2} \right\| = \dfrac{\sqrt{4^2+(-4)^2}}{2} = \dfrac{\sqrt{32}}{2} = \boxed{2\sqrt{2}}$

5. (10 points) Compute:

$$\Big[(1,2,1) \times \Big([(1,2,1) \times (3,4,1)] \times [(1,2,1) \times (5,7,9)]\Big)\Big] \times (4,10,3).$$

Hint: There is a way to do this problem without doing any computations.

Make sure to justify your answer. (If you compute all of these cross products explicitly, and you get the wrong answer, you will get at most half credit.)

$\Big[(1,2,1) \times \Big([(1,2,1)\times(3,4,1)] \times [(1,2,1)\times(5,7,9)]\Big)\Big] \times (4,10,3)$

For cross products, $(a \times b) \times c = a \times (b \times c) = (a \times c) \times b$

No No.

So then $\Big[(1,2,1) \times \Big([(1,2,1)\times(1,2,1)] \times [(3,4,1) \times (5,7,9)]\Big)\Big] \times (4,10,3)$

for any vector v, $v \times v = (0,0,0)$ so

$\Big[(1,2,1) \times \Big((0,0,0) \times [(3,4,1) \times (5,7,9)]\Big)\Big] \times (4,10,3)$

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and for any vector v, $(0,0,0) \times v = (0,0,0)$

$[(1,2,1) \times (0,0,0)] \times (4,10,3)$

$= (0,0,0)$