Math 32A/1
Fall 2016
11/14/16
Time Limit: 50 Minutes

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Day \ T.A.	Alex	David	Kevin
Tuesday	1A	1C	1E
Thursday	1B	1D	1F

This exam contains 5 pages (including this cover page) and 4 problems. Check to see if any pages are missing. Enter your name and SID number on the top of this page, cross the box correspondiing to your discussion section, and put your initials on the top of every page, in case the pages become separated. Also, have your photo ID on the desk in front of you during the exam.

Use a pen to record your answers. Calculators or computers of any kind are not allowed. You are not allowed to consult any other materials of any kind, including books, notes and your neighbors. You may use the back of this sheet for your notes ("scratch paper"). If you need additional paper, let the proctors know.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	10	8
2	10	10
3	10	10
4	10	6
Total:	40	34

Of course, if you have a question about a particular problem, please raise your hand and one of the proctors will come and talk to you.

TIM

1. (10 points) (a) Consider a particle moving in the plane:

$$\vec{r}(t) = \langle 3t - 4\sin(t), 1 - 4\cos(t)\rangle, \qquad 0 < t < 2\pi.$$

When does the particle attain its maximal speed?

(b) Find an arc length parametrization of $\vec{r}(t) = \langle -t^2, t^2, \frac{t^2}{2}\rangle$, $t \geq 0$. *Hint: Don't forget to specify the domain of your parametrization.*

a) $\vec{r}\,'(t) = \langle 3 - 4\cos(t), 4\sin(t) \rangle$

find max speed

$(3-4\cos(t))(3-4\cos(t))$

$9 - 14\cos(t) + 16\cos^2 t$

$\sqrt{(3-4\cos(t))^2 + (4\sin(t))^2}$

$\sqrt{9 - 14\cos(t) + 16\cos^2 t + 16\sin^2 t}$

$\sqrt{9 - 14\cos(t) + 16(\cos^2(t) + \sin^2(t))}$

$\sqrt{9 - 14\cos(t) + 16}$

2/4

$0 = \sqrt{25 - 14\cos(t)}$

$25 - 14\cos(t) = 0$

$\frac{-14\cos(t)}{-14} = \frac{-25}{-14}$

$\cos(t) = \frac{25}{14}$

critical point $t = \cos^{-1}\left(\frac{25}{14}\right)$

$t > \cos^{-1}\left(\frac{25}{14}\right)$

TIM

2. (10 points) (a) We saw in class that the curvature of a straight line is zero. Show the converse: if $\vec{r}(t) = \langle x(t), y(t)\rangle$ is a regular parametrization with zero curvature everywhere then the curve must be a straight line.

   (b) Consider the parametrization $\vec{r}(t) = \langle t, t^2\rangle$. Write the acceleration vector $\vec{a}(\frac{1}{2})$ as the sum of vectors parallel and normal to the direction of motion.

a) $\vec{r}(t) = \langle x(t), y(t)\rangle$

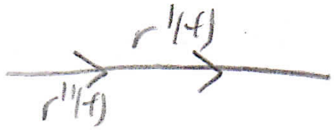
$r'(t) = \langle x'(t), y'(t)\rangle$

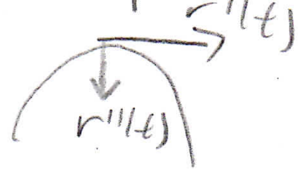
$r''(t) = \langle x''(t), y''(t)\rangle$

$$\kappa(t) = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3} = 0$$

$\|r'(t) \times r''(t)\|$ must be 0 for all t

This is only the case if the vectors $r'(t)$ and $r''(t)$ are parallel. This is only true for a straight line as for any curve other than a straight line $r''(t)$ would point inside of the curve.





is a direction to the

✓

3. (10 points) (a) Evaluate the limit or show that it does not exist:

$$\lim_{(x,y)\to(0,0)} x^2 \sin(x/y^2).$$

(b) Show that the function

$$f(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}, \qquad (x,y,z) \neq (0,0,0),$$

satisfies the Laplace equation: $f_{xx} + f_{yy} + f_{zz} = 0$.

5 a)

$$-1 \le \sin\left(\frac{x}{y^2}\right) \le 1$$

$$\lim_{(x,y)\to(0,0)} \left(-x^2 \le x^2 \sin\left(\frac{x}{y^2}\right) \le x^2 \right)$$

$$0 \le x^2 \sin\left(\frac{x}{y^2}\right) \le 0$$

$$\lim_{(x,y)\to(0,0)} x^2 \sin\left(\frac{x}{y^2}\right) = 0$$

5

$$f(x,y,z) = (x^2+y^2+z^2)^{-\frac{1}{2}}$$

b) $$f_x = -\frac{1}{2}(x^2+y^2+z^2)^{-\frac{3}{2}}(2x) = (-x)(x^2+y^2+z^2)^{-\frac{3}{2}}$$

$$f_{xx} = (-x)\,{-\frac{3}{2}}(x^2+y^2+z^2)^{-\frac{5}{2}}(2x) + (-1)(x^2+y^2+z^2)^{-\frac{3}{2}}$$

$$= 3x^2(x^2+y^2+z^2)^{-\frac{5}{2}} - (x^2+y^2+z^2)^{-\frac{3}{2}}$$

$$f_y = -\frac{1}{2}(x^2+y^2+z^2)^{-\frac{3}{2}}(2y) = (-y)(x^2+y^2+z^2)^{-\frac{3}{2}}$$

$$f_{yy} = (-y)\,{-\frac{3}{2}}(x^2+y^2+z^2)^{-\frac{5}{2}}(2y) + (-1)(x^2+y^2+z^2)^{-\frac{3}{2}}$$

4. (10 points) (a) Consider the function $f(x,y) = \frac{1}{2x+y}$. Is the vector $\langle 1, -1, -1 \rangle$ tangent to the graph $z = f(x,y)$ at the point $(\frac{1}{2}, 0, 1)$?

(b) Use linear approximation to estimate $1.01^{3.99}$. *Hint:* $1.01^{3.99}$ *is defined as* $e^{3.99\ln(1.01)}$.

a) $f(x,y) = \frac{1}{2x+y} = (2x+y)^{-1}$ point $(\frac{1}{2}, 0, 1)$

$f_x = -(2x+y)^{-2}(2) = -2(2x+y)^{-2}$ ✓

$f_y = -(2x+y)^{-2}(1) = -(2x+y)^{-2}$ ✓

$f_x(\frac{1}{2}, 0, 1) = -2(2(\frac{1}{2})+0)^{-2} = -2$

$f_y(\frac{1}{2}, 0, 1) = -(2(\frac{1}{2})+0)^{-2} = -1$ ✓

$f(\frac{1}{2}, 0, 1) = \frac{1}{2(\frac{1}{2})+0} = 1$

$z = 1 + -2(x - \frac{1}{2}) + -1(y - 0)$ ✓

$z = 1 - 2(x - \frac{1}{2}) + -y$

$z = 1 - 2x + 1 - y$

$z = -2x - y + 2$

$-1 = -2(1) - (-1) + 2$

$-1 = -2 + 1 + 2$

$-1 \neq 1$

No the vector isn't tangent to graph at this point