Math 32A - Lecture 3: Midterm 2

Released: Friday 25th February at 8am PDT
Due: Saturday 26th February by 8am PDT via Gradescope

The exam includes 6 pages (including this cover page) and 5 problems.

There are a total of 51 points available.

Please read the following instructions **very carefully** before starting the test:

• Solutions must be uploaded to Gradescope **before 8am PST on Saturday February 26th**. *Absolutely no late submissions will be accepted. I encourage you to submit your solutions several hours before the deadline to allow for possible technical complications. If you do have technical issues, please email me with a .pdf of your submission as soon as possible at the email below*

• **Please start each problem on a new page**.

• Credit will be given for fully explaining your solutions, writing clearly, legibly, and in full English sentences.

• If any questions arise during the examination, please email me: forlano@math.ucla.edu

• The exam is open book: You are permitted to use notes, texts, computers, and standard internet sites (such as Wikipedia) during the exam. You may use any theorems or results from lectures, homeworks or the midterms, without proof. However, you must verify any required assumptions in using these results.

• **The work submitted must be entirely your own:** You may not collaborate or work with anyone else to complete the exam. Students are not permitted to use online human resources such as Chegg, Math Stack Exchange, etc.

• Violations of these rules will be regarded as academic dishonesty and will be reported to the office of the Dean of Students.

• Note that it may be possible to successfully answer parts of a multi-part question without answering earlier parts.

• Good luck!

1. (1 point) Please complete a cover page with:

   - Your full name.
   - Your University ID number.
   - The following statement, along with your signature and the date:

   "I assert, on my honor, that I have not received assistance of any kind from any other person while working on this exam and that I have not used any non-permitted materials or technologies during the period of this evaluation."

2. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) = \sqrt{x^2 - y^2}$.

   (a) (2 points) Explain why the maximal domain D of f is

   $$D = \{(x, y) \in \mathbb{R}^2 \, : \, |x| \geq |y| \, \}.$$

   (b) (5 points) Explain why f is differentiable on

   $$D_0 = \{(x, y) \in \mathbb{R}^2 \, : \, |x| > |y| \, \}$$

   and compute its derivative on D_0.

   (c) (5 points) Show that $f_x(1, 1)$ does not exist and explain why f is not differentiable at $(1, 1)$.

3. Consider the space-curve $\mathbf{r}(t) = \langle e^t \cos t, e^t \sin t, e^t \rangle$ for $t \geq 0$.

   (a) (4 points) Compute the arc-length s starting at $\mathbf{r}(0)$ up to parameter $t \geq 0$.

   (b) (4 points) Compute the curvature κ as a function of the parameter $t \geq 0$.

   (c) (6 points) Find the binormal vector $\mathbf{B}(t)$ when $t = \frac{\pi}{2}$.

4. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} xy & \text{if } \ xy \geq 0, \\ \frac{x^2}{\sqrt{x^2+y^2}} & \text{if } \ xy < 0. \end{cases}$$

(a) (6 points) Show that f is continuous on $\{(x,y) \in \mathbb{R}^2 \,:\, x = 0\}$ (i.e. everywhere on the y-axis).

Hint: Given $y_0 \in \mathbb{R}$, show that $|f(x,y)| \leq |x|$ for all $(x,y) \in D_1((0,y_0))$.

(b) (6 points) Show that f is not continuous on $\{(x,y) \in \mathbb{R}^2 \,:\, x \neq 0,\ y = 0\}$ (i.e. everywhere on the x-axis except at the origin).

5. (12 points) Show that the *Gaussian solitary wave*

$$u(t,x) = e^{-\frac{1}{4}(x+\frac{1}{2}t)^2}$$

is a solution to the logarithmic Korteweg-de Vries equation

$$\frac{\partial u}{\partial t} + \frac{\partial^3 u}{\partial x^3} + \frac{\partial}{\partial x}\Big(u \log(|u|)\Big) = 0,$$

where $(t,x) \in \mathbb{R}^2$.