Math 32A - Lecture 3: Midterm 1

Released: Friday 28th January at 8am PDT
Due: Saturday 29th January by 8am PDT via Gradescope

The exam includes 7 pages (including this cover page) and 6 problems.

There are a total of 52 points available.

Please read the following instructions **very carefully** before starting the test:

• Solutions must be uploaded to Gradescope **before 8am PST on Saturday January 29th**. *Absolutely no late submissions will be accepted. I encourage you to submit your solutions several hours before the deadline to allow for possible technical complications.*

• **Please start each problem on a new page**.

• Credit will be given for fully explaining your solutions, writing clearly, legibly, and in full English sentences.

• If any questions arise during the examination, please email me: forlano@math.ucla.edu

• The exam is open book: You are permitted to use notes, texts, computers, and standard internet sites (such as Wikipedia) during the exam. You may use any theorems or results from lectures, homeworks or the midterms, without proof. However, you must verify any required assumptions in using these results.

• **The work submitted must be entirely your own:** You may not collaborate or work with anyone else to complete the exam. Students are not permitted to use online human resources such as Chegg, Math Stack Exchange, etc.

• Violations of these rules will be regarded as academic dishonesty and will be reported to the office of the Dean of Students.

• Note that it may be possible to successfully answer parts of a multi-part question without answering earlier parts.

• Good luck!

1. (1 point) Please complete a cover page with:

- Your full name.
- Your University ID number.
- The following statement, along with your signature and the date:

"I assert, on my honor, that I have not received assistance of any kind from any other person while working on this exam and that I have not used any non-permitted materials or technologies during the period of this evaluation."

2. Let $\mathbf{v} = \langle -2, \lambda, 1 \rangle$ for some scalar $\lambda \in \mathbb{R}$.

   (a) (4 points) Find the values of $\lambda \in \mathbb{R}$ such that the vector $\mathbf{v}$ has length 3.

   (b) (8 points) Find the (unique) value of λ such that the vector $\mathbf{v}$ makes an angle of $120°$ with the line

$$\mathbf{r}(t) = \langle \tfrac{1}{4}t, 2 - \tfrac{1}{2}t, 3 + \tfrac{1}{2}t \rangle, \quad t \in \mathbb{R}.$$

3. (a) (4 points) Find an equation for the plane P_1 containing the points

$$P(2,1,1), \quad Q(-1,-1,10), \quad R(1,3,-4).$$

(b) (8 points) A second plane P_2 passes through the point $(-4,2,2)$ with normal $\langle 2,-1,2\rangle$. Find a parametrisation of the line of intersection of the planes P_1 and P_2.

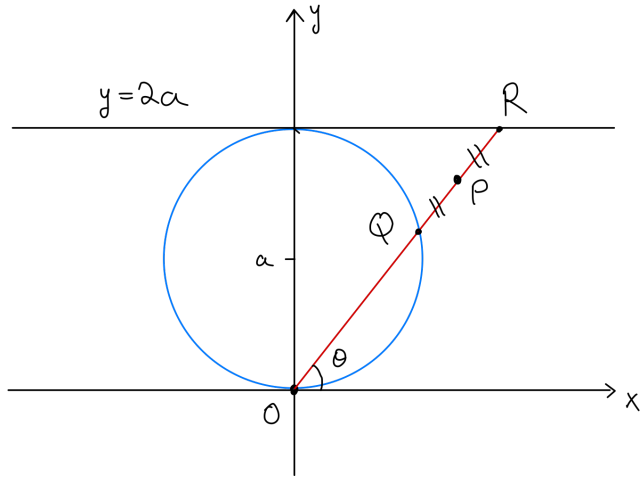
4. (10 points) Describe the x-traces, y-traces and z-traces of

$$x^2 - 2y^2 - 3z^2 + x + 2y + 3z = 0,$$

clearly labelling your plots and indicating any places where the geometry changes (if there are any). Identify the quadric surface and provide a sketch.

You do not need to compute and label axes intercepts in your trace plots.

5. Consider the figure below, where $a > 0$ and the point P is the midpoint of the line segment QR.



(a) (6 points) Show that

$$\mathbf{r}(\theta) = \langle a(1 + \sin^2\theta)\cot\theta, a(1 + \sin^2\theta)\rangle$$

for $0 < \theta < \pi$ is a parametric equation for the curve traced out by the point P in terms of the angle θ.

(b) (2 points) Show that along this curve, $y > a$ for every $x \in \mathbb{R}$ and that $y = a$ is a horizontal asymptote for the curve as $x \to +\infty$ and as $x \to -\infty$.

(c) (3 points) Using the identity

$$\sin(\tan^{-1}(u)) = \frac{u}{\sqrt{1 + u^2}},$$

find a Cartesian representation of the curve in the form $x = f(y)$, for $x > 0$.

6. (8 points) Find a parametrisation of the curve of intersection of the surfaces

$$x^2 + 2y^2 + z^2 = 5 \quad \text{and} \quad x^2 + y^2 = 1,$$

which lies in the first octant $\{(x, y, z) \in \mathbb{R}^3 : \, x, y, z \geq 0\}$.