1. (20 points) Consider the curve C parameterized by

$$\mathbf{r}(t) = \langle 3t, \cos(4t), \sin(4t) \rangle$$

for $t \geq 0$.

(a) (5 points) Calculate the unit tangent vector $\mathbf{T}$ and unit normal vector $\mathbf{N}$ to the curve C at $\mathbf{r}(t)$ for $t \geq 0$.

$r' = \langle 3, -4\sin(4t), 4\cos(4t) \rangle$

$T = \frac{r'}{\|r'\|} = \frac{\langle 3, -4\sin(4t), 4\cos(4t) \rangle}{\sqrt{25}} = \langle \frac{3}{5}, -\frac{4}{5}\sin 4t, \frac{4}{5}\cos(4t) \rangle$

$T' = \langle 0, -\frac{16}{5}\cos 4t, -\frac{16}{5}\sin(4t) \rangle$

$N = \frac{T'}{\|T'\|} = \frac{\langle 0, -\frac{16}{5}\cos(4t), -\frac{16}{5}\sin(4t) \rangle}{\frac{16}{5}} = \langle 0, -\cos(4t), -\sin(4t) \rangle$

(b) (5 points) Calculate the arc length $s(t)$ of the parameterization $\mathbf{r}(t)$ as a function of t for $t \geq 0$.

$s(t) = \int_0^t \sqrt{3^2 + 16\sin^2 4u + 16\cos^2 4u}\, du$

$s(t) = 5t$

(c) (5 points) Find the arc length parameterization $\mathbf{r}_1(s)$ of the curve C.

$t = \frac{1}{5} s$

arc length parameterization is

$\langle \frac{3}{5}s, \cos\frac{4}{5}s, \sin\frac{4}{5}s \rangle$

(d) (5 points) Calculate the curvature $\kappa(s)$ of the curve C at a point $\mathbf{r}_1(s)$.

$\kappa(s) = \left|\frac{dT}{ds}\right| = \sqrt{(\frac{3}{5})^2 + (\frac{4}{5})^2} = 1$

-4

(+1 for formula)

$z^2 - 1 = \left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2$

2. (20 points) Consider the quadratic surface given by the equation

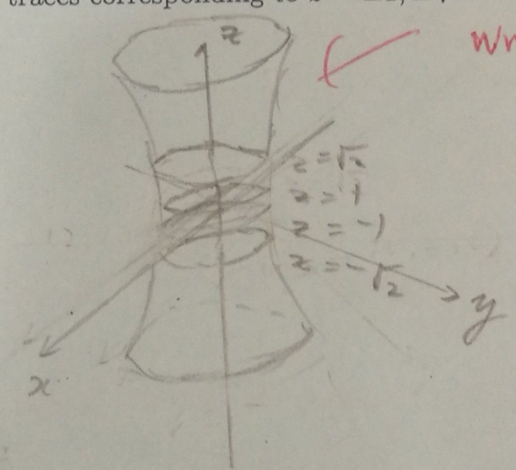
$$-x^2 - y^2 + 4z^2 = 4. \quad (1)$$

-2

(a) (2 points) Classify the surface. That is, say what type of quadratic surface is given by Equation (1).

hyperboloid (~~one~~ two sheet)

(b) (8 points) Sketch the surface and, included in your sketch, draw the horizontal traces corresponding to $z = 1$, $z = -1$, $z = \sqrt{2}$ and $z = -\sqrt{2}$. Be sure to label your axes. You are free to draw additional traces (to help you in your sketch); however, you **must** label the traces corresponding to $z = \pm 1, \pm\sqrt{2}$.



wrong graph -3

(c) (5 points) Give a parameterization $\mathbf{r}(t)$ of the trace corresponding to $z = \sqrt{2}$.

$1 = \left(\frac{x^2}{2}\right) + \left(\frac{y}{2}\right)^2$

$\vec{r}(t) = < \ldots >$

$y = \pm\sqrt{4 - \left(\frac{x}{2}\right)^2}$ -1

(d) (5 points) Using your parameterization, calculate the curvature $\kappa(t)$ of the trace corresponding to $z = \sqrt{2}$ at $\mathbf{r}(t)$.

$\kappa(t) = \frac{|f''(x)|}{(1 + f'(x)^2)^{\frac{3}{2}}} = \frac{1}{2}$ -2.

Show work.

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3. (20 points) Define a function $f : \mathbb{R}^2 \to \mathbb{R}$ by

$$f(x,y) = \begin{cases} \frac{x}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases} \qquad \text{for } (x,y) \in \mathbb{R}^2.$$

(a) (8 points) Determine the set of points at which f is continuous. Justify your answer.

$\{x,y \mid (x,y) \neq 0,\ (x,y) \in \mathbb{R}^2\}$

we can tell that the function is continuous when the denominator is not 0. Why?

7

at 0 limit doesn't exist since we approach from x-axis

$\lim_{(x,y)\to(0,0)} f(x,0) = \lim_{x\to 0} \frac{1}{x}$ which DNE

(rational function)

so all point in $\mathbb{R}^2$ except $(0,0)$

(b) (10 points) Compute the partial derivatives f_x and f_y where they exist.

$f_x = \frac{2x - 2x^3}{(x^2+y^2)^2}$

$f_y = -2xy(x^2+y^2)^{-2}$

+3

(c) (2 points) Are the partial derivatives f_x and f_y continuous at all points in $\mathbb{R}^2$?

f_x continuous in all $\mathbb{R}^2$ except $(x,y)=(0,0)$ since f_x doesn't exist there.

1

f_y is continuous in all $\mathbb{R}^2$ except $(x,y)=(0,0)$ since f_y doesn't exist there.

4. (20 points) In what follows, f is a two-variable function with domain $\mathcal{D} \subseteq \mathbb{R}$. You may assume that f is defined near $(0,0)$.

(a) (10 points) TRUE OR FALSE (circle one, 2 points each)

True	False	According to Kepler's laws, planets travel in ellipses with the sun at one focus.
True	False	The curvature κ of a curve C is *always* non-negative.
True	False	To verify that the limit $\lim_{(x,y)\to(0,0)} f(x,y)$ exits and is equal to L, it suffices to show that $f(x,y)$ tends to L as (x,y) approaches $(0,0)$ along all lines of the form $y = mx$.
True	False	For f to be continuous at $(0,0)$, it is necessary that $(0,0) \in \mathcal{D}$.
True	False	Contour lines corresponding to distinct z-values of f can *never* intersect.

-2

(b) (5 points) Determine whether the following limit exists and, if it does, compute it. Justify your answer.

$$\lim_{(x,y)\to(0,0)} 4x^2y^4 \cos\left(\frac{1}{x^4+y^2}\right)$$

$-4x^2y^4 < 4x^2y^4 \cos\left(\frac{1}{x^2+y^2}\right) < 4x^2y^4$

$\lim_{(x,y)\to(0,0)} -4x^2y^2 = 0 \qquad \lim_{(x,y)\to(0,0)} 4x^2y^4 = 0$

according to squeeze theorem

$\lim_{(x,y)\to(0,0)} 4x^2y^2 \cos\left(\frac{1}{x^4+y^2}\right) = 0$

(c) (5 points) Consider the function

$$f(x,y) = x^2y + \frac{\cos\left(\frac{x^4}{x^2+1}\right)}{2+\sin(x^2)}$$

defined for all $(x,y) \in \mathbb{R}^2$. Calculate $f_{xy} = \frac{\partial^2 f}{\partial y \partial x}$ at the point (a,b) (You may assume that f_{xy} and f_{yx} exist and are continuous on $\mathbb{R}^2$).

$f_y(x,y) = x^2$

$f_{xy}(x,y) = 2x$

since f_{xy} and f_{yx} exsit and are continuous

according to clauraidt theorem

$f_{xy} = f_{yx} = 2x \quad f_{xy}$ at (a,b) is $2a$