MATH 32A: Calculus of Several Variables
Lecture 4, Fall 2018
Midterm exam 2

University of California, Los Angeles

Instructor: David Arnold

Total time: 40 minutes

Question	Marks	Score
1	6	4
2	5	5
3	6	4
4	3	2
Total:	20	15

First Name:

Last Name:

ID number:

Signature:

Date: 11-19-18

By signing this form you declare that you will act in accordance with the UCLA Student Conduct Code.

Please circle your discussion section

	Zach	Patrick	Bertrand
Tuesday	4A	4C	4E
Thursday	4B	4D	4F

Instructions

- Calculators without programming/computer algebra capabilities may be used
- Notes/textbooks may not be used
- Students must have photo ID to prove identity
- Mobile phones must be switched off, and placed in closed bags on the floor
- If extra space for answers is required, there is a blank page at the end of the exam booklet

1. Short answer questions. Show brief working, or give a short justification of your answer. Unexplained answers receive no credit.

-2 (a) (2 marks) Consider two different parameterisations $\mathbf{r}_1(t_1)$ and $\mathbf{r}_2(t_2)$ of the same space curve. Imagine using $\mathbf{r}_1$ to calculate the curvature $\kappa_1(t_1)$ and $\mathbf{r}_2$ to calculate the curvature $\kappa_2(t_2)$. Will $\kappa_1(t_1)$ always, or ever, be equal to $\kappa_2(t_2)$? Explain your answer.

$K_1(t_1)$ will be equal to $K_2(t_2)$ if $r_1(t_1)$ and $r_2(t_2)$ have the same speed, since curvature is the derivative of the unit tangent vector T with respect to t, where T is $\frac{r'(t)}{\|r'(t)\|}$, and $\|r'(t)\|$ is speed.

You also need the positions are the same

(b) (2 marks) Explain why $f(x, y) = \sin(x)e^{-y}$ is continuous everywhere, or give a point where it is discontinuous.

$f(x,y) = \sin(x)e^{-y}$ is continuous everywhere since it is the composition of the functions $f(x) = \sin(x)$ and $f(y) = e^{-y}$ which are continuous at all x and y, respectively.

(c) (1 mark) Evaluate $\frac{\partial}{\partial x}\left(x\sin(x^2y) + y^2\right)$.

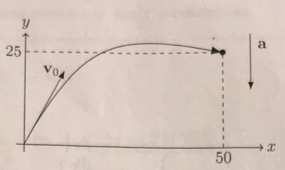
$x(\cos(x^2y)(2xy)) + \sin(x^2y)$

$= 2x^2y\cos(x^2y) + \sin(x^2y)$

(d) (1 mark) True or False? For an object following a path $\mathbf{r}(t)$, the Normal vector $\mathbf{N}(t)$ is perpendicular to the acceleration vector $\mathbf{a}$?

False. $N(t)$ is perpendicular to $\underset{\sim}{a}$ if and only if $a \cdot \underset{\sim}{N} = 0$, and $a \cdot \underset{\sim}{N}$ is equal to a_N, the component of $\underset{\sim}{a}$ perpendicular to velocity $\underset{\sim}{r}'(t)$. a_N is not equal to 0 as long as the object following path $r(t)$ changes direction and its speed $\|r'(t)\|$ is not equal to 0.

2. (5 marks) An object is fired with initial velocity $\mathbf{v}_0$ from starting position $\mathbf{x}_0 = (0, 0)$ at time $t = 0$. At time $t = 5$ it hits a target at position $(50, 25)$. The acceleration of the object is $(0, -10)$ and is constant. Find the objects position at time t in terms of $\mathbf{v}_0$, and hence determine the initial velocity of the object.



$a(t) = (0, -10)$

$v(t) = \int a(t)\,dt = (0, -10t) + \underset{\sim}{c}_0$

$v_0 = v(0) = (0, -10(0)) + \underset{\sim}{c}_0$

$v_0 = \underset{\sim}{c}_0$

$x(t) = \int v(t)\,dt = (0, -5t^2) + v_0 t + \underset{\sim}{c}_1$

$x_0 = (0,0) = (0, -5(0)^2) + v_0(0) + \underset{\sim}{c}_1$

$\underset{\sim}{c}_1 = (0,0)$

$x(t) = (0, -5t^2) + v_0 t$

$x(5) = (50, 25) = (0, -5(5)^2) + v_0(5)$

$(50, 25) = (0, -125) + v_0(5)$

$v_0(5) = (50, 150)$

$\boxed{v_0 = (10, 30)}$ ✓

continued on next page

3. Consider

$$f(x,y) = \begin{cases} \dfrac{|x|y}{\sqrt{x^2+y^2}} & (x,y) \neq (0,0), \\ 0 & (x,y) = (0,0). \end{cases}$$

(a) (4 marks) Determine whether or not f is continuous.

-2

Continuous if $\lim_{(x,y)\to(0,0)} \frac{|x|y}{\sqrt{x^2+y^2}} = 0$

$$\lim_{(x,y)\to(0,0)} \frac{|x|y}{\sqrt{x^2+y^2}} = \lim_{r\to 0} \frac{|r\cos\theta| r\sin\theta}{r}$$

$$= \lim_{r\to 0} |r\cos\theta| \sin\theta$$

$$= 0$$

Since $\lim_{(x,y)\to(0,0)} f(x,y) = f(0,0)$, $f(x,y)$ is continuous

need to check $(x,y) \neq (0,0)$ as well

(b) (2 marks) Use the limit definition of the derivative to calculate $f_x(0,0)$.

$$f_x(0,0) = \lim_{h\to 0} \frac{f(0+h,0) - f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{|h|(0)}{\sqrt{h^2+0^2}} - 0}{h}$$

$$= \lim_{h\to 0} \frac{0}{h}$$

continued on next page

4. (3 marks) Draw a contour map of $f(x,y) = 1/(1+\sqrt{x^2+y^2})$. Give the equations and describe the shapes of the level curves.

2/3

these
are actually
empty
...
ust keep
$< z \leq 1$

$z=-1$:

$\frac{1}{1+\sqrt{x^2+y^2}} = -1$

$1 = -1 - \sqrt{x^2+y^2}$

$\sqrt{x^2+y^2} = 2$

$x^2+y^2 = 4$

circle with radius 2

~~$z=1$:~~

~~$\frac{1}{1+\sqrt{x^2+y^2}} = 1$~~

~~$1 = 1+\sqrt{x^2+y^2}$~~

~~$\sqrt{x^2+y^2} = 0$~~

~~$x^2+y^2 = 0$~~

~~$z=2$:~~

$\frac{1}{1+\sqrt{x^2+y^2}} = 2$

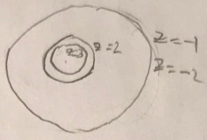
$1 = 2 + 2\sqrt{x^2+y^2}$

$-1 = 2\sqrt{x^2+y^2}$

$\sqrt{x^2+y^2} = -\frac{1}{2}$

$x^2+y^2 = \frac{1}{4}$

circle with radius $\frac{1}{2}$



$z=-2$:

$\frac{1}{1+\sqrt{x^2+y^2}} = -2$

$1 = -2 - 2\sqrt{x^2+y^2}$

$2\sqrt{x^2+y^2} = 1$

$\sqrt{x^2+y^2} = -\frac{1}{2}$

$x^2+y^2 = \frac{1}{4}$

circle with radius $\frac{1}{4}$

$z=3$

radius $= \left(\frac{1-3}{3}\right)^2$

$= \left(\frac{-2}{3}\right)^2$

$= \frac{4}{9}$

$\frac{1}{1+\sqrt{x^2+y^2}} = z$

$1 = z + z\sqrt{x^2+y^2}$

$\sqrt{x^2+y^2} = \frac{1-z}{z}$ → level curves are circles with radius $= \left(\frac{1-z}{z}\right)^2$, undefined at $z=0$

continued on next page

This page left blank intentionally, use for extra answer space if required.

End of exam