MATH 32A: Calculus of Several Variables
Lecture 4, Fall 2018
Midterm exam 2

University of California, Los Angeles

Instructor: David Arnold

Total time: 40 minutes

Question	Marks	Score
1	6	6
2	5	5
3	6	6
4	3	3
Total:	20	20

First Name: Richard

Last Name: Cheng

ID number: 805083331

Signature: Richard Cheng

Date: 11/19/2018

By signing this form you declare that you will act in accordance with the UCLA Student Conduct Code.

Please circle your discussion section

	Zach	Patrick	Bertrand
Tuesday	4A	4C	$\boxed{4E}$
Thursday	4B	4D	4F

Instructions

- Calculators without programming/computer algebra capabilities may be used
- Notes/textbooks may not be used
- Students must have photo ID to prove identity
- Mobile phones must be switched off, and placed in closed bags on the floor
- If extra space for answers is required, there is a blank page at the end of the exam booklet

1. Short answer questions. Show brief working, or give a short justification of your answer. Unexplained answers receive no credit.

   (a) (2 marks) Consider two different parameterisations $\mathbf{r}_1(t_1)$ and $\mathbf{r}_2(t_2)$ of the same space curve. Imagine using $\mathbf{r}_1$ to calculate the curvature $\kappa_1(t_1)$ and $\mathbf{r}_2$ to calculate the curvature $\kappa_2(t_2)$. Will $\kappa_1(t_1)$ always, or ever, be equal to $\kappa_2(t_2)$? Explain your answer.

   $k_1(t_1)$ will sometimes be equal to $k_2(t_2)$.
   For example, any parameterisation of a straight line will have a constant curvature of 0.
   However, different parameterisations of a parabola can result in different equations for the curvature when $t_1 \neq t_2$.

   (b) (2 marks) Explain why $f(x, y) = \sin(x)e^{-y}$ is continuous everywhere, or give a point where it is discontinuous.

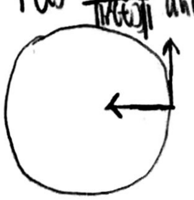
   Because $\sin(x)$ is continuous everywhere and e^{-y} is continuous everywhere, $f(x,y) = \sin(x)e^{-y}$ is also continuous everywhere.

   (c) (1 mark) Evaluate $\frac{\partial}{\partial x}\left(x\sin(x^2y) + y^2\right)$.

   $$\frac{\partial}{\partial x}\left(x\sin(x^2y)+y^2\right)$$
   $$= 1\sin(x^2y) + x\cos(x^2y)(2xy) + 0$$
   $$= \boxed{\sin(x^2y) + 2x^2y\cos(x^2y)}$$

   (d) (1 mark) True or False? For an object following a path $\mathbf{r}(t)$, the Normal vector $\mathbf{N}(t)$ is perpendicular to the acceleration vector $\mathbf{a}$?

   False. Since $a = r''(t)$, $T(t) = \frac{r'(t)}{\|r'(t)\|}$ and $N(t) = \frac{T'(t)}{\|T'(t)\|}$, the Normal vector will be in the direction of the acceleration vector, not perpendicular to it.



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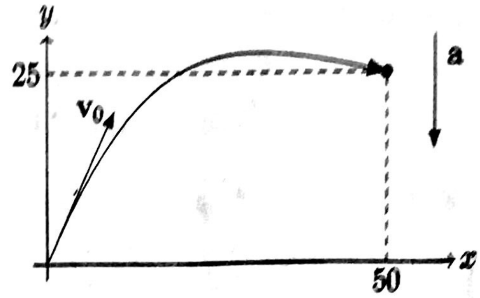
2. (5 marks) An object is fired with initial velocity $\mathbf{v}_0$ from starting position $\mathbf{x}_0 = (0,0)$ at time $t = 0$. At time $t = 5$ it hits a target at position $(50, 25)$. The acceleration of the object is $(0, -10)$ and is constant. Find the objects position at time t in terms of $\mathbf{v}_0$, and hence determine the initial velocity of the object.



$a(t) = (0, -10)$

$v(t) = \int a(t)\,dt$

$= (C_1, -10t + C_2)$

$v(0) = (C_1, C_2)$

$x(t) = \int v(t)\,dt$

$= (C_1 t + C_3, -5t^2 + C_2 t + C_4)$

$= (C_3, C_4) + t(C_1, -5t + C_2)$

$= (C_3, C_4 - 5t^2) + t v_0$

$x(0) = (0, 0)$

$\boxed{x(t) = (0, -5t^2) + t v_0}$

$x(5) = (0, -125) + 5 v_0 = (50, 25)$

$5 v_{0x} = 50 \Rightarrow v_{0x} = 10$

$5 v_{0y} - 125 = 25 \Rightarrow v_{0y} = 30$

$\boxed{v_0 = (10, 30)}$ ✓

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3. Consider

$$f(x,y) = \begin{cases} \dfrac{|x|y}{\sqrt{x^2+y^2}} & (x,y) \neq (0,0), \\ 0 & (x,y) = (0,0). \end{cases}$$

(a) (4 marks) Determine whether or not f is continuous.

At $(x,y) \neq (0,0)$, $f(x,y)$ is continuous because it's a ratio of two continuous functions that are not equal to 0.

$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{|x|y}{\sqrt{x^2+y^2}}$

We let $x = r\cos\theta$ and $y = r\sin\theta$.

$\lim_{r\to 0} \frac{|r\cos\theta| r\sin\theta}{\sqrt{r^2\cos^2\theta + r^2\sin^2\theta}} = \lim_{r\to 0} \frac{r^2|\cos\theta|\sin\theta}{\sqrt{r^2}} = \lim_{r\to 0} r|\cos\theta|\sin\theta = 0$

Since $\lim_{(x,y)\to(0,0)} f(x,y) = f(0,0) = 0$, $f(x,y)$ is continuous at $(0,0)$.

Therefore, f is continuous.

(b) (2 marks) Use the limit definition of the derivative to calculate $f_x(0,0)$.

$\lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{\frac{|h|\cdot 0}{\sqrt{h^2+0^2}} - 0}{h}$

$= \lim_{h\to 0} \frac{\frac{|h|\cdot 0}{|h|} - 0}{h}$

$= \lim_{h\to 0} \frac{0-0}{h} = 0$

$\boxed{f_x(0,0) = 0}$

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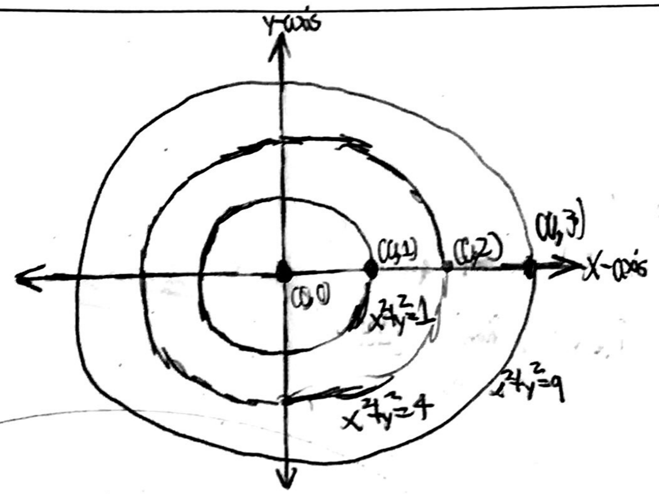
4. **(3 marks)** Draw a contour map of $f(x,y) = 1/(1+\sqrt{x^2+y^2})$. Give the equations and describe the shapes of the level curves.



$f(x,y)=1 \Rightarrow \frac{1}{1+\sqrt{x^2+y^2}}=1 \Rightarrow \sqrt{x^2+y^2}=0 \Rightarrow (x,y)=(0,0)$

$f(x,y)=\frac{1}{2} \Rightarrow \frac{1}{1+\sqrt{x^2+y^2}}=\frac{1}{2} \Rightarrow \sqrt{x^2+y^2}=1 \Rightarrow x^2+y^2=1$

$f(x,y)=\frac{1}{3} \Rightarrow \sqrt{x^2+y^2}=2 \Rightarrow x^2+y^2=4$

good!

$f(x,y)=\frac{1}{1+r^2} \Rightarrow x^2+y^2=r^2$

$1+r^2=\frac{1}{f(x,y)}$

The level curves are circles centered at the origin of radius $\sqrt{\frac{1}{f(x,y)}-1}$.

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