19F-MATH32A-2 Midterm 1

[redacted]

TOTAL POINTS

46 / 50

QUESTION 1

1 1 (a) **5 / 5**

- ✓ **- 0 pts** Correct
- **- 1 pts** Arithmetic Error

QUESTION 2

2 1 (b) **4 / 5**

- **- 0 pts** Correct
- ✓ **- 1 pts** Arithmetic Mistake
- **- 2.5 pts** Somewhat Correct Procedure
- **- 5 pts** Incorrect
- **- 5 pts** Blank/No Answer

QUESTION 3

3 1 (c) **5 / 5**

- ✓ **- 0 pts** Correct
- **- 1 pts** Arithmetic Error
- **- 0.5 pts** Mildly Incorrect Justification
- **- 1 pts** Incorrect Justification
- **- 2.5 pts** No Justification
- **- 5 pts** Incorrect
- **- 5 pts** Blank/No Answer

QUESTION 4

4 2 **10 / 10**

- ✓ **+ 3 pts** Correctly obtained two vectors from the three points
- ✓ **+ 4 pts** Computed cross product for coefficients of plane equation
- ✓ **+ 3 pts** Correctly found translation constant
- **- 1 pts** Arithmetic mistake (even if inconsequential)
- **- 3 pts** Gave expression for a line
- **+ 0 pts** No credit due

QUESTION 5

5 3 **10 / 10**

- ✓ **- 0 pts** Correct
- **- 1 pts** Small mistake in derivative
- **- 2 pts** Multiple mistakes in derivative
- **- 3 pts** No derivative calculation
- **- 1 pts** Mistake in evaluating derivative at pi/2
- **- 2 pts** Did not evaluate derivative at pi/2
- **- 1 pts** Mistake in evaluating position at pi/2
- **- 2 pts** Did not evaluate position at pi/2
- **- 1 pts** Small mistake in tangent line
- **- 2 pts** Multiple mistakes in tangent line
- **- 3 pts** Did not write tangent line

QUESTION 6

6 4 **10 / 10**

- **+ 0 pts** Incorrect
- **+ 1 pts** Find r'(t)
- **+ 2 pts** correct arc length formula
- **+ 1 pts** Have correct bounds of intergration
- **+ 1 pts** Factor out t^2 in square root
- **+ 2 pts** Correct u substitution
- **+ 2 pts** Finish integral correctly. should have (1/27)*sqrt(4+9t^2) or equivalent expression
- **+ 1 pts** plug in correct upper and lower bounds
- ✓ **+ 10 pts** Correct answer: (13^(3/2) - 8)/27
- **+ 1 pts** Done some correct integration
- **- 1 pts** Minor arithmetic error

QUESTION 7

7 5 **2 / 5**

- ✓ **+ 1 pts** State r(t).r(t) = 4
- **+ 1 pts** Differentiate both sides
- **+ 1 pts** Use product rule correctly
- **+ 1 pts** Use commutativity: r(t).r'(t) = r'(t).r(t)
- ✓ **+ 1 pts** Show that you know dot product = 0 means orthogonal
- **+ 0 pts** Start problem with r(t).r(t) = 4

Midterm 1

Last Name: [redacted]

First Name:

Student ID:

Signature:

Instructions: Do not open this exam until instructed to do so. You will have 50 minutes to complete the exam. Please print your name and student ID number above, and circle the number of your discussion section. **You may use a calculator, as long as it is not a graphing calculator.** You may not use books, notes, or any other material to help you. Please make sure **your phone is silenced** and stowed where you cannot see it. You may use any available space on the exam for scratch work. If you need more scratch paper, please ask one of the proctors. You must **show your work** to receive credit. Please circle or box your final answers.

Please do not write below this line.

Question	Points	Score
1	15	
2	10	
3	10	
4	10	
5	5	
Total:	50	

1. For all sub-parts of this question, let $\mathbf{u} = \langle 3, -9, 5\rangle$ and $\mathbf{v} = \langle 3, 0, 4\rangle$.

(a) (5 points) Compute $\mathbf{u} - 2\,\mathbf{v}$

$\langle 3,-9,5\rangle - 2\langle 3,0,4\rangle$

$\langle 3,-9,5\rangle - \langle 6,0,8\rangle = \boxed{\langle -3,-9,-3\rangle}$

(b) (5 points) Write $\mathbf{u}$ as a sum: $\mathbf{u} = \mathbf{u}_{\parallel \mathbf{v}} + \mathbf{u}_{\perp \mathbf{v}}$ where $\mathbf{u}_{\parallel \mathbf{v}}$ is parallel to $\mathbf{v}$ and $\mathbf{u}_{\perp \mathbf{v}}$ is perpendicular to $\mathbf{v}$.

$\vec{e}_{\vec{v}} = \frac{1}{\sqrt{3^2+0^2+4^2}}\langle 3,0,4\rangle = \langle \frac{3}{5}, 0, \frac{4}{5}\rangle$

$\vec{u}\cdot\vec{e}_{\vec{v}} = \langle 3,-9,5\rangle \cdot \langle \frac{3}{5}, 0, \frac{4}{5}\rangle = \frac{9}{5} + 0 + \frac{20}{5} = \frac{29}{5}$

$\frac{29}{5}\langle \frac{3}{5}, 0, \frac{4}{5}\rangle = \boxed{\langle \frac{87}{25}, 0, \frac{116}{25}\rangle = u_{\parallel v}}$

$u_{\perp v} = \langle 3, -9, 5\rangle - \langle \frac{87}{25}, 0, \frac{116}{25}\rangle$

$= \langle \frac{75}{25}, -9, \frac{25}{5}\rangle - \langle \frac{87}{25}, 0, \frac{116}{25}\rangle$

$= \boxed{\langle \frac{-12}{25}, -9, -\frac{91}{25}\rangle = u_{\perp v}}$

$\boxed{\langle 3,-9,5\rangle = \langle \frac{87}{25}, 0, \frac{116}{25}\rangle + \langle -\frac{12}{25}, -9, -\frac{91}{25}\rangle}$

(c) (5 points) Let $P = (5, 11, -3)$ and $Q = (8, 11, 1)$. Is the vector $\overrightarrow{PQ}$ equivalent to either $\mathbf{u}$ or $\mathbf{v}$? Justify your answer.

$\overrightarrow{PQ} = \langle 8-5, 11-11, 1--3\rangle = \langle 3,0,4\rangle$

$\overrightarrow{PQ}$ is equivalent to $\vec{v}$, since both have the exact same compounts.

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2. (10 points) Find the equation of the plane passing through the points $P = (1,2,1)$, $Q = (2,2,4)$ and $R = (-1,2,3)$

$\vec{PQ} = \langle 2-1, 2-2, 4-1 \rangle = \langle 1, 0, 3 \rangle$

$\vec{RQ} = \langle 2--1, 2-2, 4-3 \rangle = \langle 3, 0, 1 \rangle$

$\vec{PQ} \times \vec{RQ} = \begin{vmatrix} i & j & k \\ 1 & 0 & 3 \\ 3 & 0 & 1 \end{vmatrix} = i(0-0) - j(1-9) + k(0-0)$
$= \langle 0, 8, 0 \rangle = \vec{n}$

$0x + 8y + 0z = d$

$d = \vec{OP_0} \cdot \vec{n} = \langle 1, 2, 1 \rangle \cdot \langle 0, 8, 0 \rangle = 16$

$\boxed{8y = 16}$

3. (10 points) Find a parametrization of the tangent line to the curve given by $\mathbf{r}(t) = \langle 3\cos(t), 5\sin(t), 4\cos(t)\rangle$ at the point $t = \pi/2$

$r'(t) = \langle -3\sin(t), 5\cos(t), -4\sin(t)\rangle$

$r'(\pi/2) = \langle -3\sin(\frac{\pi}{2}), 5\cos(\frac{\pi}{2}), -4\sin(\frac{\pi}{2})\rangle$

$= \langle -3, 0, -4\rangle$

$r(\pi/2) = \langle 3\cos(\frac{\pi}{2}), 5\sin(\frac{\pi}{2}), 4\cos(\frac{\pi}{2})\rangle$

$= \langle 0, 5, 0\rangle$

$\ell(s) = \langle 0, 5, 0\rangle + s\langle -3, 0, -4\rangle$

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4. (10 points) Find the arc length, from $t = 0$ to $t = 1$ of the curve with parametrization $\mathbf{r}(t) = \langle t^2, t^3, 1\rangle$.

$$s = \int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\, dt$$

$$= \int_0^1 \sqrt{(2t)^2 + (3t^2)^2 + (0)^2}\, dt$$

$$= \int_0^1 \sqrt{4t^2 + 9t^4}\, dt$$

$$= \int_0^1 \sqrt{t^2(4+9t^2)}\, dt$$

$$= \int_0^1 t\sqrt{4+9t^2}\, dt$$

$u = 4+9t^2$

$du = 18t\, dt$

$\frac{1}{18} du = t\, dt$

$$= \frac{1}{18}\int_4^{13} u^{1/2}\, du$$

$u(1) = 4+9 = 13$

$u(0) = 4+0 = 4$

$$= \frac{1}{18_9}\left[\frac{2}{3} u^{3/2}\right]_4^{13}$$

$$= \frac{1}{27}\left[(13)^{3/2} - (4)^{3/2}\right]$$

$v = \langle 3, 0, 4 \rangle$

5. (5 points) Show that if $\| \mathbf{r}(t) \| = 2$ then $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ are orthogonal. *(Hint: It might help you to recall that for any vector* $\mathbf{v}$ *we have that* $\| \mathbf{v} \|^2 = \mathbf{v} \cdot \mathbf{v}$*)*

if $\|r(t)\| = 2$; $4 = r(t) \cdot r(t)$

$\sqrt{(x(t))^2 + (y(t))^2 + (z(t))^2} = 2$

if $\|r(t)\| = 2$: has to be a circle with radius 2, tangent lines to circles are always orthogonal to path

ex: $r(t) = \langle \sqrt{2}\cos(t), \sqrt{2}\sin(t) \rangle$: $\|r(t)\| = \sqrt{(\sqrt{2}\cos(t))^2 + (\sqrt{2}\sin(t))^2}$

$= \sqrt{4} = 2$ ✓

$r'(t) = \langle -\sqrt{2}\sin(t), \sqrt{2}\cos(t) \rangle$

$r(t) \cdot r'(t) = \langle \sqrt{2}\cos(t), \sqrt{2}\sin(t) \rangle \cdot \langle -\sqrt{2}\sin(t), \sqrt{2}\cos(t) \rangle$

$= (-2\sin(t)\cos(t) + 2\sin(t)\cos(t)) = 0$, thus orthogonal.