Math 32A - Winter 2019
Exam 2

Full Name: Sadhana Vadrevu

UID: 205095030

Circle the name of your TA and the day of your discussion:

Qi Guo   Talon Stark   Tianqi (Tim) Wu

Tuesday   Thursday

Instructions:

- Read each problem carefully.
- Show all work clearly and circle or box your final answer where appropriate.
- Justify your answers. A correct final answer without valid reasoning will not receive credit.
- Simplify your answers as much as possible.
- Include units with your answer where applicable.
- Calculators are not allowed but you may have a 3×5 inch notecard.

Page	Points	Score
1	32	
2	26	
3	22	
4	20	
Total:	100	

1. (20 points) Consider the surface defined by $ze^{2x} + x^2y + y = 4 + 2e^{z+1}$.

(a) Find an equation of the tangent plane to the surface at the point $P = (0, 6, -1)$.

$\vec{n}$ of tangent plane $= \nabla F$

$F = ze^{2x} + x^2y + y - 2e^{z+1} = 4$

$\nabla F = \langle 2ze^{2x} + 2xy,\ x^2 + 1,\ e^{2x} - 2e^{z+1} \rangle$ @ $P = (0, 6, -1)$

$\nabla F = \langle 2(-1)\cancel{e^{2(0)}}^{1} + \cancel{2(0)(6)}^{0},\ 0^2 + 1,\ \cancel{e^{2(0)}}^{1} - 2\cancel{e^{-1+1}}^{1} \rangle$

$= \langle -2, 1, -1 \rangle$

$-2(x-0) + 1(y-6) - 1(z+1) = 0$

$-2x + y - z = 6 + 1$

$\boxed{-2x + y - z = 7}$

(b) Find a vector equation for the line passing through the surface at $P = (0, 6, -1)$ orthogonal to the plane found in part (a).

orthogonal to plane $= \vec{n}$

$\vec{n} = \langle -2, 1, -1 \rangle$ point $= \langle 0, 6, -1 \rangle$

$\therefore \boxed{\mathcal{L}(t) = \langle -2t, 6 + t, -1 - t \rangle}$

2. (12 points) Either give an example of a function $f(x, y)$ with $f_x(x, y) = 2x + y^2e^x$ and $f_y(x, y) = x^2 + y^2e^x$ or show that no such function f can exist.

$f_{xy} = 2e^x y$ $f_{yx} = 2x + y^2e^x$

$f_{xy} \neq f_{yx} \therefore$ function does not satisfy Clairaut's Theorem so function cannot exist

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3. (16 points) Reparameterize the curve $\mathbf{r}(t) = \langle \sqrt{3}\,t^2, \cos(t^2), \sin(t^2) \rangle$ where $t \geq 0$ with respect to arc length.

$$s = \int_0^t \|\vec{r}\,'(t)\|\,dt = \int_0^t 4t\,dt = 2t^2\Big|_0^t = 2t^2$$

$$\vec{r}\,'(t) = \langle 2\sqrt{3}\,t,\ -\sin(t^2)\cdot 2t,\ \cos(t^2)\cdot 2t \rangle$$

$$\|\vec{r}\,'(t)\| = \sqrt{(4(3)t^2) + 4t^2\sin^2 t^2 + 4t^2\cos^2 t^2}$$

$$= \sqrt{12t^2 + 4t^2} = \sqrt{16t^2} = 4t$$

$$s = 2t^2 \Rightarrow t = \sqrt{\tfrac{s}{2}}$$

$$\boxed{\vec{r}_1(s) = \left\langle \frac{s\sqrt{3}}{2},\ \cos\left(\frac{s}{2}\right),\ \sin\left(\frac{s}{2}\right) \right\rangle}$$

$$= \vec{r}\left(\sqrt{\tfrac{s}{2}}\right)$$

4. (10 points) Show the following limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2 + 2y^4}$$

path $y = x$: $\displaystyle\lim_{x\to 0} \frac{x(x^2)}{x^2+2x^4} = \lim_{x\to 0} \frac{x^3}{x^2+2x^4} = \lim_{x\to 0} \frac{x}{1+2x^2} = 0$

path $x = y^2$: $\displaystyle\lim_{y\to 0} \frac{(y^2)(y^2)}{y^4+2y^4} = \lim_{y\to 0} \frac{\cancel{y^4}}{3\cancel{y^4}} = \frac{1}{3}$ $\quad 0 \neq \frac{1}{3}$ ∴ limit DNE ✓

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5. (22 points) Consider the function $f(x,y) = \sqrt{10 - x^2 - 5y^2}$.

(a) Use a linear approximation to $f(x,y)$ at the point $(2,1)$ to estimate the value of $f(2.05, 0.97)$.

linear approximation:

$f(a+\Delta x, b+\Delta y) \approx f(a,b) + f_x(a,b)\Delta x + f_y(a,b)\Delta y$

$(a,b) = (2,1)$

$f(2,1) = \sqrt{10-4-5} = \sqrt{1} = 1$

$f_x = \frac{1}{\cancel{2}}(10-x^2-5y^2)^{-1/2} \cdot -\cancel{2}x \Rightarrow f_x(2,1) = \frac{-2}{\sqrt{10-4-5}} = -2$

$f_y = \frac{1}{\cancel{2}}(10-x^2-5y^2)^{-1/2} \cdot -\cancel{10}^{5}y \Rightarrow f_y(2,1) = \frac{-5}{\sqrt{10-4-5}} = -5$

$f(2+\Delta x, 1+\Delta y) \approx 1 - 2\Delta x - 5\Delta y$

$\Delta x = 0.05, \quad \Delta y = -0.03$

$f(2.05, 0.97) \approx 1 - 2(0.05) - 5(-0.03) = 1 - 0.1 + 0.15 = \boxed{1.05}$

(b) Find the directional derivative of f at the point $(2,1)$ in the direction of $\langle 4,3 \rangle$.

$\vec{u} = \frac{\langle 4,3 \rangle}{\sqrt{16+9}} = \left\langle \frac{4}{5}, \frac{3}{5} \right\rangle \qquad \nabla f = \langle \underset{f_x(2,1)}{\underset{\uparrow}{-2}}, \underset{f_y(2,1)}{\underset{\uparrow}{-5}} \rangle$

$D_{\vec{u}} f = \nabla f \cdot \vec{u}$

$= \langle -2, -5 \rangle \cdot \left\langle \frac{4}{5}, \frac{3}{5} \right\rangle = -\frac{8}{5} - 3 = -\frac{8}{5} - \frac{15}{5} = \boxed{\frac{-23}{5}}$

(c) Find the maximum rate of change of f at the point $(2,1)$.

maximum $= \|\nabla f\| = \sqrt{4+25} = \boxed{\sqrt{29}}$

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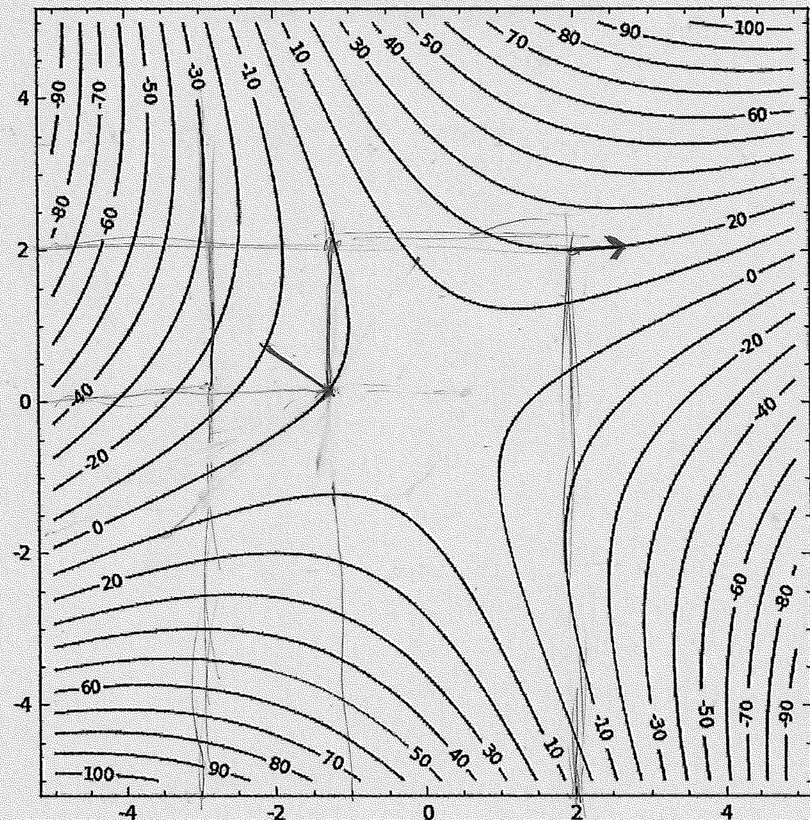
6. (20 points) Consider the contour plot for $f(x, y)$ below.



(a) Determine the sign of each of the following derivatives.

$f_x(-3, 0)$ ___+___ $f_y(-3, 0)$ ___−___

$f_{xx}(-3, 0)$ ___−___ $f_{xy}(-3, 0)$ ___+___ $f_{yy}(-3, 0)$ ___+___

(b) Give the components of a unit vector in the direction of the steepest decline at the point $(-1, 0)$. (You may estimate as necessary.)

steepest decline direction = opposite of ∇f direction = how to get to lower elevation fastest $\langle f_x, f_y \rangle$

$$\boxed{\left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle}$$

(c) Give the components of a unit vector orthogonal to $\nabla f(2, 2)$. (You may estimate as necessary.)

unit vector orthogonal to $\nabla f \Rightarrow \underline{D_{\vec{u}} f = 0}$

↳ height doesn't change

$$\boxed{\langle 1, 0 \rangle}$$

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You may use this page for scratch work. Work found on this page will not be graded unless clearly indicated in the exam.