1. (15 points) Find the equation of the plane that contains the following two lines.

$$\mathbf{r}(t) = \langle 2+t, 2+3t, 3+t \rangle$$
$$\mathbf{s}(t) = \langle 5+t, 15+5t, 10+2t \rangle.$$

Intersection:

$r(t) = \langle 2+t, 2+3t, 3+t \rangle$

$S(P) = \langle 5+P, 15+5p, 10+2p \rangle$

x: $2+t=5+p$

y: $2+3t=15+5p$

z: $3+t=10+2p$

x: $2+t=5+p$

$t=p+3$

y: $2+3(p+3)=15+5p$

$2+3p+9=15+5p$

$11=15+2p$

$-4=2p$

$p=-2$

$t=p+3$

$t=1$

Point of Intersection: $(3,5,4)$

Direction vectors

$r(t) = \langle 1,3,1 \rangle$

$s(t) = \langle 1,5,2 \rangle$

Normal vector

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ 1 & 3 \\ 1 & 5 \end{matrix}$$

$6\vec{i} + \vec{j} + 5\vec{k} - 2\vec{j} - 5\vec{i} - 3\vec{k}$

$\vec{i} - \vec{j} + 2\vec{k}$

$\langle 1,-1,2 \rangle$

Equation using pt of intersection

$(x-3)-(y-5)+2(z-4)=0$

2. (15 points) Describe in words and sketch a picture of the region in $\mathbb{R}^3$ represented by the following inequality. In addition to sketching the region in $\mathbb{R}^3$, sketch the $x = 0$ trace.

$$1 \leq y^2 + \frac{z^2}{4} \leq 4.$$

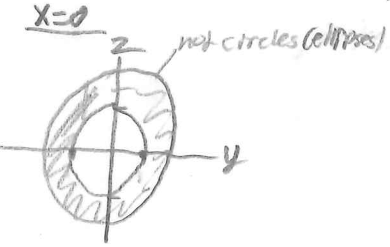
$1 \leq y^2 + \frac{z^2}{4}$

$\frac{y^2}{4} + \frac{z^2}{16} \leq 1$

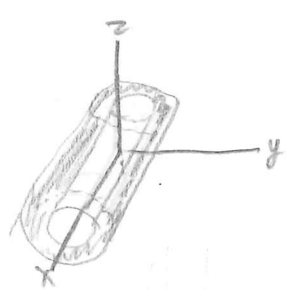
Words

This diagram is a thickened elliptic cylinder because it traces an ellipse of the y-z plane.

X=0



$\mathbb{R}^3$



sorry for horrible drawing!

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3. (22 points) Consider the points $P = (-1, 8, 3)$, $Q = (2, 3, 0)$ and $R = (2, 5, -2)$.

(a) The points P, Q, and R form a triangle. Which type of triangle is it? Circle one and justify your answer.

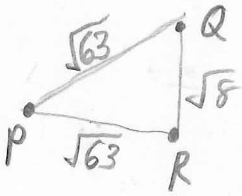
i. Isosceles

ii. Right

iii. Isosceles and right

iv. Equilateral

v. None of the above



$\vec{PQ} = \langle 2+1, 3-8, 0-3 \rangle$

$\vec{PQ} = \langle 3, -5, -3 \rangle$

$||\vec{PQ}|| = \sqrt{3^2+5^2+3^2} = \sqrt{63}$

$\vec{PR} = \langle 2+1, 5-8, -2-3 \rangle$

$\vec{PR} = \langle 3, -3, 5 \rangle$

$||\vec{PR}|| = \sqrt{3^2+3^2+5^2} = \sqrt{63}$

$\vec{QR} = \langle 2-2, 5-3, -2-0 \rangle$

$\vec{QR} = \langle 0, 2, -2 \rangle$

$||\vec{QR}|| = \sqrt{0^2+2^2+2^2} = \sqrt{8}$

It forms an isoceles triangle because two of the side lengths have a magnitude of $\sqrt{63}$, while the final side has a side length of $\sqrt{8}$ as shown in the work and diagram above

(b) Determine whether the points P, Q, R, and $S = (-1, 9, 5)$ all lie on a plane.

$\vec{PQ} = \langle 3, -5, -3 \rangle$

$\vec{PR} = \langle 3, -3, 5 \rangle$

Normal

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -5 & -3 \\ 3 & -3 & 5 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ 3 & -5 \\ 3 & -3 \end{matrix}$$

$-25\vec{i} - 9\vec{j} - 9\vec{k} - 15\vec{j} + 9\vec{i} + 15\vec{k}$

$-16\vec{i} - 24\vec{j} + 6\vec{k}$

$\vec{n} = \langle -16, -24, 6 \rangle$

$P(-1, 8, 3)$

Equation

$-16(x+1) - 24(y-8) + 6(z-3) = 0$

Plug in $S(-1, 9, 5)$

$-16(0) - 24(1) + 6(2) = 0$

$-24 + 12 \neq 0$ so that don't lie on same plane

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4. (15 points) Find parametric equations for the tangent line to the curve defined by $\mathbf{r}(t) = \langle 2\ln t, 6\sqrt{t}, t^2 \rangle$ at $t = 1$.

$r(1) = \langle 2\ln 1, 6\sqrt{1}, 1^2 \rangle$

$r(1) = \langle 0, 6, 1 \rangle$

$r'(t) = \langle \frac{2}{t}, \frac{6}{2\sqrt{t}}, 2t \rangle$

$r'(1) = \langle 2, 3, 2 \rangle$

$\vec{L}(t) = \langle 0, 6, 1 \rangle + t\langle 2, 3, 2 \rangle$

$x = 2t$
$y = 6 + 3t$
$z = 1 + 2t$

5. (15 points) Let $A = (-2, 0, 1)$ and $B = (0, 4, 5)$. Find the set of all points $P = (x, y, z)$ such that $\overrightarrow{AP}$ is orthogonal to $\overrightarrow{BP}$. Give a precise geometric description of your answer.

$\overrightarrow{AP} = \langle x+2, y, z-1 \rangle$ $\quad \overrightarrow{AP} \cdot \overrightarrow{BP} = 0$

$\overrightarrow{BP} = \langle x, y-4, z-5 \rangle$

$\langle x+2, y, z-1 \rangle \cdot \langle x, y-4, z-5 \rangle = 0$

$x(x+2) + y(y-4) + (z-1)(z-5) = 0$

$x^2 + 2x + y^2 - 4y + z^2 - 6z + 5 = 0$

Complete the square

$x^2 + 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 5 + 4 = 0 + 1 + 4 + 4$

$(x+1)^2 + (y-2)^2 + (z-3)^2 = 9$

The set of all points is a sphere with center $(-1, 2, 3)$ and radius 3

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6. (18 points) Match each vector function with its space curve.

(a) $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$ ___C___

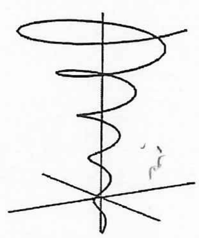
(b) $\mathbf{r}(t) = \langle t, \sin t, \overset{\cos t}{\cancel{\sin t}} \rangle$ ___D___

(c) $\mathbf{r}(t) = \langle 3 - 2t, 3 - 2t, 1 + t \rangle$ ___F___

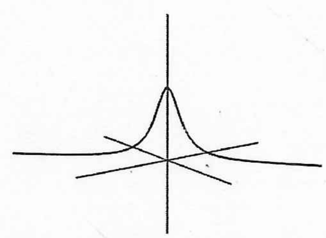
(d) $\mathbf{r}(t) = \langle e^{0.1t} \cos t, e^{0.1t} \sin t, t \rangle$ ___A___

(e) $\mathbf{r}(t) = \langle \cos t, \sin t, \cos(4t) \rangle$ ___E___

(f) $\mathbf{r}(t) = \left\langle t, -t, \dfrac{1}{1+t^2} \right\rangle$ ___B___



(A)



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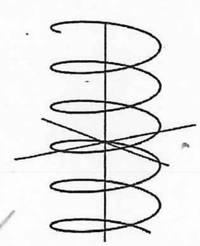

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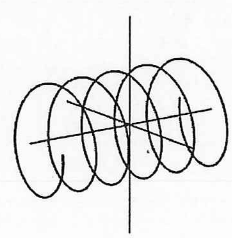

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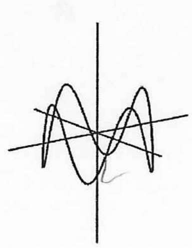

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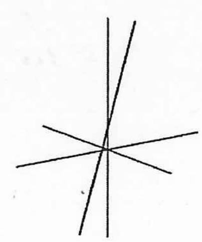

(F)

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You may use this page for scratch work. Work found on this page will not be graded unless clearly indicated in the exam.