Math 32A - Fall 2018

Exam 1

Full Name: ______________________

UID: ______________________

Circle the name of your TA and the day of your discussion:

Kevin Carlson Weiyi Liu Sang Truong

Tuesday Thursday

Instructions:

- Read each problem carefully.
- Show all work clearly and circle or box your final answer where appropriate.
- Justify your answers. A correct final answer without valid reasoning will not receive credit.
- Simplify your answers as much as possible.
- Include units with your answer where applicable.
- Calculators are not allowed but you may have a 3×5 inch notecard.

Page	Points	Score
1	20	20
2	24	24
3	16	13
4	20	14
5	20	17
Total:	100	88

1. (3 points) True or False? For any two vectors $\mathbf{a}$ and $\mathbf{b}$ in $\mathbb{R}^3$, $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{a}$.

(a) True. (b) False.

$a \times b = -b \times a$

2. (3 points) True or False? For any three vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ in $\mathbb{R}^3$, if $\mathbf{a} \cdot \mathbf{b} = 0$ and $\mathbf{b} \cdot \mathbf{c} = 0$, then $\mathbf{a} \cdot \mathbf{c} = 0$.

$a \cdot b = 0$ $\quad b \cdot c = 0$

(a) True. (b) False.

$a \cdot c = 0$

if $a = c$ then $a \cdot a = \|a\|^2$

3. Consider the points $P = (-2, 2, 0)$, $Q = (0, 1, -1)$ and $R = (-1, 2, -2)$.

(a) (10 points) Find the equation of the plane containing the points P, Q, and R.

$\vec{PQ} = \langle 0+2, 1-2, -1-0 \rangle = \langle 2, -1, -1 \rangle$

$\vec{PR} = \langle -1+2, 2-2, -2-0 \rangle = \langle 1, 0, -2 \rangle$

$$\begin{vmatrix} i & j & k \\ 2 & -1 & -1 \\ 1 & 0 & -2 \end{vmatrix} = i(2-0) - j(-4+1) + k(0+1) = \langle 2, 3, 1 \rangle$$

$4 \quad -6$

$2(x+2) + 3(y-2) + 1(z-0) = 0$

$2x + 3y + z = 2$

(b) (4 points) Find the area of the triangle with vertices P, Q, and R.

$$\frac{\|\vec{PQ} \times \vec{QR}\|}{2} = \frac{\|\langle 2, 3, 1 \rangle\|}{2} = \frac{\sqrt{4+9+1}}{2} = \frac{\sqrt{14}}{2}$$

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4. Consider the lines

$$\mathbf{r}(t) = \langle 1+t, 1-t, 2t\rangle$$
$$\mathbf{q}(t) = \langle 2-t, t, 4\rangle$$

(a) (8 points) Find the point of intersection of the two lines.

$r(t) = x = 1+t_r \quad q(t) = x = 2-t_q \qquad 1+t_r = 2-t_q \qquad 3 = 2-t_q \quad t_q = -1$

$y = 1-t_r \qquad y = t_q \qquad 1-t_r = t_q \qquad -1 = t_q$

$z = 2t_r \qquad z = 4 \qquad 2t_r = 4$

$t_r = 2$

$t_r = 2 \quad t_q = -1$

$r(2) = \langle 3, -1, 4\rangle \qquad$ at point $(3, -1, 4\rangle$

$q(-1) = \langle 3, -1, 4\rangle$

(b) (8 points) Find the equation of the plane that contains the two lines.

$$\vec{n}_r \times \vec{n}_q = \langle 1, -1, 2\rangle \times \langle -1, 1, 0\rangle = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ -1 & 1 & 0 \end{vmatrix} = i(0-2) - j(0+2) + k(1-1)$$

$\langle -2, -2, 0\rangle$ normal

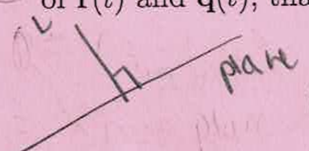
$(3, -1, 4)$ point

$-2(x-3) - 2(y+1) + 0(z-4) = 0$

$-2x - 2y = -4 \qquad 2x + 2y = 4$

$x + y = 2$

(c) (8 points) Find parametric equations for the line through the point of intersection of $\mathbf{r}(t)$ and $\mathbf{q}(t)$, that is perpendicular to the plane containing $\mathbf{r}(t)$ and $\mathbf{q}(t)$.



$p = (3, -1, 4) \quad \langle -2, -2, 0\rangle$ = plane direction vector

$r(t) = \langle 3, -1, 4\rangle + \langle -2, -2, 0\rangle t$

$r(t) = \langle 3-2t, -1-2t, 4\rangle$

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$z = \cos(-y)$

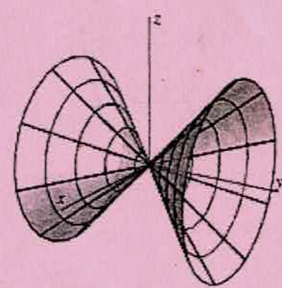
5. (16 points) Matching: Choose the picture that each equation describes. (*Hint:* Consider traces.)

$x=0$ $\quad$ $x^2 - z^2 = y$ $\quad$ Open at y

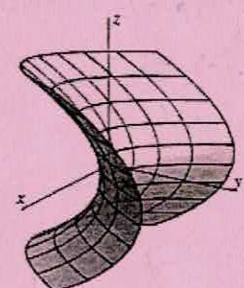
(a) $z = \cos(x - y)$ ~~G~~ D $\quad$ (b) $x^2 - y - z^2 = 0$ B $\quad$ (c) $x^2 - y + z^2 = 1$ F

(d) $x^2 - y^2 + z^2 = 0$ A $\quad$ (e) $x^2 - y^2 + z^2 = -1$ I

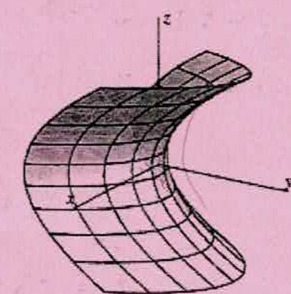
$x^2 + z^2 = y^2$ $\quad$ $1 = y^2 - x^2 - z^2$



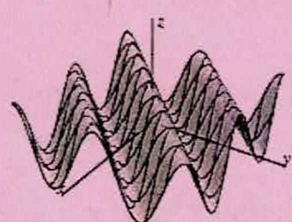
(A)



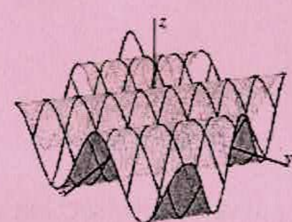
(B)



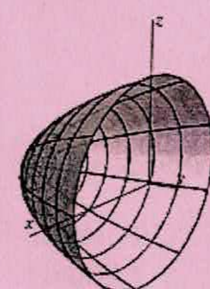
(C)



(D)

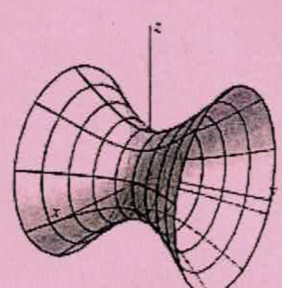


(E)

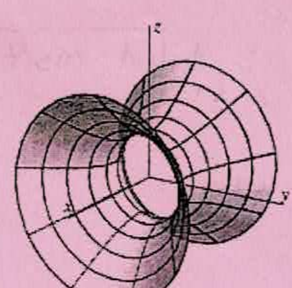


(F)

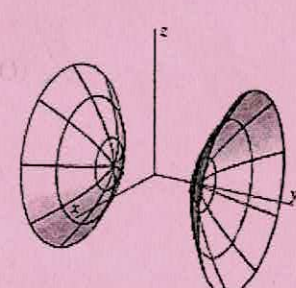
-3



(G)



(H)



(I)

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$Q=(0,1,1)$

6. (10 points) Find the point on the plane $2x-3y-z=-7$ that is closest to the point $(7,-2,-1)$. $=P$

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$\vec{PQ}=\lambda\langle 2,-3,-1\rangle$ ✓

$2(2\lambda+7)-3(-3\lambda-2)-(-\lambda-1)=-7$ ✓

$7-Q_x=2\lambda$ $Q_x=2\lambda+7$ $7-2\lambda$

$-2-Q_y=-3\lambda$ $Q_y=-3\lambda-2$ $-2+3\lambda$

$-1-Q_z=-1\lambda$ $Q_z=-\lambda-1$ $-1+\lambda$

$\cancel{4\lambda}+14\cancel{-4\lambda}+6\cancel{+\lambda}+1=-7$

$-4\lambda=-7-21$

$-4\lambda=-28$

$\cancel{\lambda=7}$

$\vec{PQ}=7\langle 2,-3,-1\rangle$

$\vec{PQ}=\langle 14,-21,-7\rangle$

$Q_x=21$

$Q_y=-23$

$Q_z=-8$

$Q=(21,-23,-8)$

7. (10 points) Parametrize the curve of intersection of the ellipsoid $x^2+y^2+4z^2=4$ with $y\geq 0$ and the circular cylinder $x^2+z^2=1$.

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$x=\cos t$ ~~$y=\sin t$~~

$z^2=1-x^2$

$z^2=1-\cos^2 t$

$y\geq 0$

$\sin t\geq 0$

$\langle \cos t, \sin t, \cancel{\pm\sqrt{1-\cos^2 t}}\rangle$ $0<t<\pi$

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8. 8. (10 points) Find parametric equations for the tangent line to the curve defined by $\mathbf{r}(t) = \langle 4\cos t, t, 2\sin t \rangle$ at $t = \pi$.

$\vec{r}'(t) = \langle 4\sin t, 1, 2\cos t \rangle$

$\vec{L}(t) = \vec{r}(\pi) + \vec{r}'(\pi)$

$\vec{r}'(\pi) = \langle 0, 1, -2 \rangle$ (-1) notation

$\vec{r}(\pi) = \langle -4, \pi, 0 \rangle$

$\vec{L}(t) = \langle -4, \pi, 0 \rangle + \langle 0, 1, -2 \rangle t$

$= \langle -4, \pi + t, -2t \rangle$ ✓

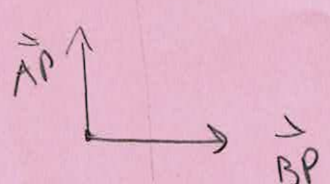
(-1) $\begin{cases} x = \cdots \\ y = \cdots \\ z = \cdots \end{cases}$

9. 9. (10 points) Let $A = (-4, 0, 1)$ and $B = (0, 2, 3)$. Find the set of all points $P = (x, y, z)$ such that $\overrightarrow{AP}$ is orthogonal to $\overrightarrow{BP}$. Give a precise geometric description of your answer.

$\vec{AP} = \langle x+4, y, z-1 \rangle$ ✓ $\quad \vec{BP} = \langle x, y-2, z-3 \rangle$ $\quad (z-3)(z-1)$

$z^2 - 3z - z + 3$

$\overrightarrow{AP} \cdot \overrightarrow{BP} = 0$ orthogonal



$x^2 + 4x + y^2 - 2y + z^2 - 4z + 3 = 0$

$(x^2 + 4x + 4) + (y^2 - 2y + 1) + (z^2 - 4z + 3) = 0$

$(x+2)^2 + (y^2 - 1)^2 + (z \qquad = 4 + 1$

$(x+2)^2 + (y^{\cancel{2}} - 1)^2 + (z^2 - 4z + 4) = 6$

$(x+2)^2 + (y^{\cancel{2}} - 1)^2 + (z^{\cancel{2}} - 2)^2 = 6$

$\frac{(x+2)^2}{6} + \frac{(y-1)^2}{6} + \frac{(z^{\cancel{2}} - 2)^2}{6} = 1$

(-1) miscalculate ~~ellipse~~ sphere with radius $\sqrt{6}$ with center $(-2, 1, \cancel{+}2)$

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