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MATH 32A Winter 2017
Midterm 1

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This exam contains 7 pages (including this page) and 5 questions. Total of points is 100.
Work neatly and show all your work, including all intermediate steps and also explain in words how you are solving a problem. Partial credits are available for most problems. Calculators are not allowed in this exam. You have 50 minutes.

Grade Table (for instructor use only)

Question	Points	Score
1	20	20
2	20	13
3	20	8
4	20	20
5	20	20
Total:	100	81

21

1. (20 points) Let $\vec{v}$ be the vector $(1,2,3)$ and $\vec{w}$ be the vector $(1,1,0)$
   (a) (6 points) Find the unit vector in the same direction as $\vec{v}$;
   (b) (6 points) Find the angle between the vectors $\vec{v}$ and $\vec{w}$;
   (c) (8 points) Let l_1 be the line through $(0,0,0)$ in the direction of $\vec{v}$ and l_2 be the line through $(-1,0,3)$ in the direction of $\vec{w}$. Write down the parametric equations of l_1 and l_2. Then determine if these two lines are intersecting.

1) a) $\vec{e}_v = \frac{\vec{v}}{\|\vec{v}\|} = \frac{\langle 1,2,3\rangle}{\sqrt{1^2+2^2+3^2}} = \frac{\langle 1,2,3\rangle}{\sqrt{14}} = \boxed{\left\langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right\rangle}$

b.) $\cos\theta = \frac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\|\vec{w}\|} = \frac{\langle 1,2,3\rangle \cdot \langle 1,1,0\rangle}{(\sqrt{14})(\sqrt{2})} = \frac{1+2}{\sqrt{28}} = \frac{3}{\sqrt{28}}$

$\boxed{\theta = \arccos\left(\frac{3}{\sqrt{28}}\right)}$

c.) $\boxed{\begin{array}{l} l_1(t) = t(1,2,3) \\ l_2(t) = (-1,0,3) + t(1,1,0) \end{array}}$

$l_1(t) = (t, 2t, 3t)$ $\quad l_2(t) = (-1+t, t, 3)$

$t_1 = -1 + t_2$ $\quad 2t_1 = t_2$ $\quad 3t_1 = 3$

$1 = 1$ ✓ $\quad t_2 = 2$ $\quad t_1 = 1$

$2 = 2$ ✓ $\quad 3 = 3$ ✓

$\boxed{\text{The lines intersect}}$

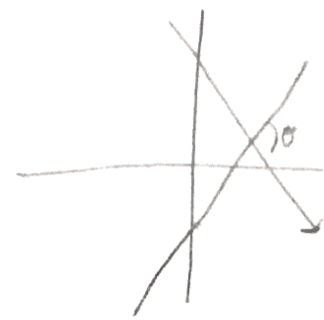
2. (20 points) (a) (10 points) Find the acute angle between the lines $2x - y = 3$ and $3x + y = 7$.

(b) (10 points) Under what conditions is the cross product of two nonzero vectors $\vec{v}$ and $\vec{w}$ equal to the zero vector, i.e., when is $\vec{v} \times \vec{w} = \vec{0}$ where $\vec{v}, \vec{w} \neq \vec{0}$.

a.) $y_1 = 2x - 3$ $\quad y_2 = -3x + 7$

$y_1 = \langle 2, 1\rangle + t\langle 1, 2\rangle \quad y_2 = \langle 2, 1\rangle + t\langle 1, -3\rangle$

let $t = 2$ $\quad y_1 = \langle 4, 5\rangle \quad y_2 = \langle 4, -5\rangle$



$$\cos\theta = \frac{16 - 25}{16 + 25} = \frac{-9}{41}$$

$$\boxed{\theta = \pi - \arccos\left(-\frac{9}{41}\right)}$$

$2x - 3 = -3x + 7$

$5x = 10$

$x = 2$

3

b.) when $\vec{v}$ & $\vec{w}$ are parallel

OR when $\vec{v} = \lambda\vec{w}$

3. (20 points) Let $\vec{u} = (0, 1, -1)$ and $\vec{v} = (2, 1, 2)$. Find a vector $\vec{w}$ such that

   (a) $\vec{w} = \lambda\vec{v}$ for some scalar λ and

   (b) $\vec{u} - \vec{w} \perp \vec{v}$.

   Draw a picture illustrating the relations between $\vec{u}$, $\vec{v}$ and $\vec{w}$.

$\boxed{\vec{w} = \langle \lambda 2, \lambda, \lambda 2 \rangle}$

$\vec{w} \perp \vec{v}$

$\vec{w} \perp \vec{v} = \langle 0, 0, 0 \rangle$

b/c $\vec{w}$ and $\vec{v}$ are parallel

$\langle -2\lambda, 1-\lambda, -1-2\lambda \rangle \perp (2,1,2)$

$\langle \lambda 2, \lambda, \lambda 2 \rangle = \langle 0, 1, -1 \rangle - \vec{w} \perp \vec{v}$

$\vec{w} \perp \vec{v} = \langle -\lambda 2, 1-\lambda, -1-\lambda 2 \rangle$

$\vec{w} = r\vec{v} + \vec{n}$

$\langle 0, 0, 0 \rangle = \langle -\lambda 2, 1-\lambda, \cancel{-\lambda-\lambda 2} \rangle$

$\vec{w} \perp \vec{v} = \langle 0, 0, 0 \rangle$

there is no $\vec{w}$ b/c there is no λ that would make this true

4. (20 points) Find the area of the parallelogram with vertices $A = (-2, 1)$, $B = (0, 4)$, $C = (4, 2)$ and $D = (2, -1)$.

$$\text{Area} = \|\vec{AB} \times \vec{AD}\|$$

$$= \|\langle 2, 3\rangle \times \langle 4, -2\rangle\|$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 0 \\ 4 & -2 & 0 \end{vmatrix}$$

$$= \|\langle 0, 0, (-4-12)\rangle\|$$

$$= \|\langle 0, 0, 16\rangle\|$$

$$= \sqrt{16^2} = \boxed{16}$$

5. (20 points) Find the volumn of the parallelpiped spanned by $\vec{AB}$, $\vec{AC}$ and $\vec{AD}$ where the points are $A = (1, 1, 1)$, $B = (2, 0, 3)$, $C = (4, 1, 7)$ and $D = (3, -1, -2)$.

$\vec{AB} = \langle 1, -1, 2 \rangle \quad \vec{AC} = \langle 3, 0, 6 \rangle \quad \vec{AD} = \langle 2, -2, -3 \rangle$

$$\text{Volume} = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & 6 \\ 2 & -2 & -3 \end{vmatrix}$$

$$= \left| 1(12) + 1(-9-12) + 2(-6) \right|$$

$$= \boxed{21}$$