1. Let

$$f(x) = \frac{x+1}{x-1}.$$

a). Find the domain and the range of $f(x)$. [3 points]

b). Justify that $f(x)$ is one-to-one. Find the inverse function $f^{-1}(x)$. [4 points]

c). Find the domain and the range of $f^{-1}(x)$. [2 points]

d). Find the derivatives $f'(x)$ and $(f^{-1}(x))'$. [6 points]

$x = \frac{y+1}{y-1}$

$xy - x = y + 1$

$xy - y = x + 1$

$y(x-1) = x+1 \quad \lim_{x\to\infty} \frac{x+1}{x-1} = 1$

$\lim_{x\to-\infty} = 1$

A. Domain $= (-\infty, 1) \cup (1, \infty)$

Range $(-\infty, 1) \cup (1, \infty)$

B. $\frac{dy}{dx} = \frac{(x-1)(1) - (x+1)(1)}{(x-1)^2} \quad \frac{\cancel{x}-1 - (\cancel{x}+1)}{(x-1)^2} = \frac{-2}{(x-1)^2}$

$\frac{dy}{dx}$ is always negative on domain $(-\infty, 1) \cup (1, \infty)$

so $f(x)$ is strictly decreasing and is one-to one

$y = \frac{x+1}{x-1}$

$y(x-1) = x+1$

$yx - y = x + 1$

$yx - x = y + 1$

$x(y-1) = y+1$

$x = \frac{y+1}{y-1}$

$f^{-1}(x) = \frac{x+1}{x-1}$

doesnt work because f, f' not continuous

C.) Domain $= (-\infty, 1) \cup (1, \infty)$

Range $= (-\infty, 1) \cup (1, \infty)$

D. $f'(x) = \frac{(x-1)(1) - (x+1)(1)}{(x-1)^2} \quad \frac{x-1-x-1}{(x-1)^2} = f'(x) = \frac{-2}{(x-1)^2}$

$(f^{-1}(x))' = \frac{(x-1)(1) - (x+1)(1)}{(x-1)^2} \quad (f^{-1}(x))' = \frac{-2}{(x-1)^2}$

$2(5x)\cdot 5$

$2\ln x =$ $\quad \log_3 1 = 0 \quad 3, 1$

$(1, 0)$

$(2, .390)$

2. Let $\frac{2}{x}$ $(5x)^2$ $10x$

$$f(x) = (\log_3(x))^2.$$

a). Sketch the graph of $f(x)$. [3 points]

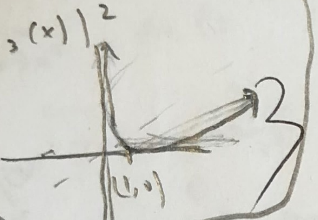
b). Determine a domain on which f^{-1} exists. Sketch the graph of f^{-1}. [6 points]

c). Find the derivative $f'(x)$. [3 points] $\quad \frac{\ln x}{x\ln 3}$

d). Evaluate the integral

$$\int_3^9 \frac{\log_3(x)}{x}dx. \quad [3 \text{ points}]$$

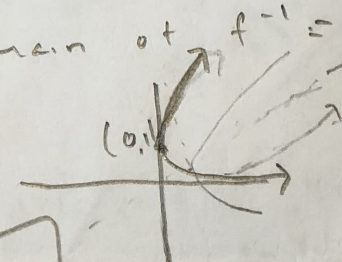
A. $(\log_3(x))^2$



$(1,0)$

B.) Domain of $f^{-1}(x)$ = Range of $f(x)$

Domain of $f^{-1} = (0, \infty)$



$(0,1)$

6

C. $f(x) = (\log_3 x)^2$

$f(x) = \left(\frac{\ln x}{\ln 3}\right)^2$

$f'(x) = 2\left(\frac{\ln x}{\ln 3}\right)\cdot\left(\frac{1}{x\ln 3}\right)$

$((\log_3 x))^2$

$\left(\frac{\ln x}{\ln 3}\right)^2$

$\int (\ln x)(\ln x)$

$u = \ln x$

$\frac{(\ln 9)^2}{2} - \frac{(\ln 3)^2}{2}$

$\int (\log_3(x))^2$

$\left(\frac{\ln x}{\ln 3}\right)^2 = \left(\frac{1}{\ln 3}\right)^2 \cdot (\ln x)^2$

$\left(\frac{1}{\ln 3}\right)^2 \int (\ln x)^2$

$v = x \quad dv = 1$

D. $\int_3^9 \frac{\log_3 x}{x} = \int_3^9 \frac{\frac{\ln x}{\ln 3}}{x} =$

$u = \ln x$

$du = \frac{1}{x}dx$

$\ln 3\left(\frac{u^2}{2}\Big|_3^9\right)$

$\int_3^9 \frac{\ln x}{x\ln 3}$??

$u = \ln x$

$du = \frac{1}{x}dx$

$dx = du(x)$

$\ln 3\left(\frac{(\ln x)^2}{2}\Big|_3^9\right)$

$\ln(3)\int_3^9$

3. Let

$$f(x) = x^{1/x^2} \quad \text{for } x > 0.$$

5 a). Calculate

$$\lim_{x\to 0^+} f(x) \quad \text{and} \quad \lim_{x\to\infty} f(x). \quad \text{[6 points]}$$

5 b). Find the maximum value of $f(x)$, and determine the intervals on which $f(x)$ is increasing and decreasing. [6 points]

$e^{\ln(x^{1/x^2})}$ $\quad \ln y = \ln x^{1/x^2}$

$e^{\ln x}$ $\quad \ln y = \frac{\ln x}{x^2}$

$\log x^8 + \log A^5$

$\frac{dy}{dx} \cdot y = \frac{(x^2)(\frac{1}{x}) - (\ln x)(2x)}{x^4}$

$x - \frac{(2x)(\ln x)}{x^4}$

a. $\lim_{x\to 0^+} x^{1/x^2}$

$\lim_{x\to 0^+} e^{\ln(x^{1/x^2})}$

$\lim_{x\to 0^+} \frac{\ln(x)}{x^2} = 0$

$e^{-\infty} = 0$

$\boxed{\lim_{x\to 0^+} f(x) = 0}$ 3

$\lim_{x\to 0^+} \frac{\ln x}{x^2} = \frac{-\infty}{\text{small number}} =$

$\lim_{x\to 0^+} \frac{\ln x}{x^2} = -\infty$

Please be much more organized

$y = x^{1/x^2}$

$\ln y = \frac{\ln x}{x^2}$

$\frac{1}{y} \cdot \frac{dy}{dx} = \cancel{\frac{(\ln x)(2x) - (\ln x)(2x)}{x^4}}$

$\lim_{x\to\infty} x^{1/x^2}$

$\lim_{x\to\infty} e^{\ln x^{1/x^2}}$

$\lim_{x\to\infty} e^{\frac{\ln x}{x^2}}$

$e^0 = 1$

$\lim_{x\to\infty} \frac{\ln x}{x^2} = \frac{\infty}{\infty}$

$\frac{\frac{1}{x}}{2x} = \frac{1}{x} \cdot \frac{1}{2x} =$

$\lim_{x\to\infty} \frac{1}{2x^2} = 0$

2 $\boxed{\lim_{x\to\infty} f(x) = \cancel{0}}$

$\cancel{\frac{(x^2)(\frac{1}{x}) - (\ln x)(2x)}{(x^4)}}$

$\frac{dy}{dx} \cdot y = \ln x$

$\frac{dy}{dx} = \left(\frac{x - (\ln x)(2x)}{x^4}\right)\left(x^{1/x^2}\right)$

b.) $f(x) = x^{1/x^2}$

$\ln(y) = \ln(x^{1/x^2})$

$\ln y = \frac{\ln x}{x^2} \rightarrow \ln x (x^{-2})$

$\frac{1}{y}\frac{dy}{dx} = \frac{(x^2)(\frac{1}{x}) - (\ln x)(2x)}{x^4}$

5 $\quad 0 = \left(\frac{x - (\ln x)(2x)}{x^4}\right)$

$x - (\ln x)(2x) = 0$

$x = \sqrt{e}$

why is this a max?

why?

Maximum value 1.202 at $x = \sqrt{e}$

Increasing $(0, \sqrt{e})$

Decreasing $(\sqrt{e}, \infty)$

2 4. a). Find the value of $tan^{-1}(cos(2\pi))$. [2 points]

3 b). Evaluate the integral

$$\int_1^{\sqrt{3}} \frac{dx}{(tan^{-1}(x))(1+x^2)}. \quad [5 \text{ points}]$$

$\frac{1}{4}$

A. $\cos(2\pi) = 1$

$\tan^{-1}(1) = \pi/4$

B. $\int_1^{\sqrt{3}} \frac{dx}{(\tan^{-1}(x))(1+x^2)}$

$= \int_1^{\sqrt{3}} \frac{1}{u}\, du$

$u = \tan^{-1}(x)$

$du = \frac{1}{1+x^2}\, dx$

$dx = \frac{du}{1+x^2}$

$= \ln|u| \Big|_1^{\sqrt{3}} \cancel{dx}$

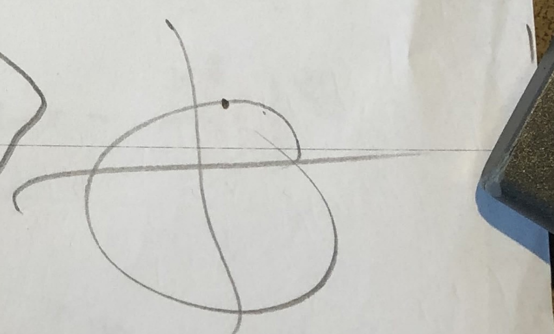
$= \ln(\tan^{-1}(x)) \Big|_1^{\sqrt{3}}$

$\ln(\pi/3) - \ln(\frac{\pi}{4})$

?

not a 3

=



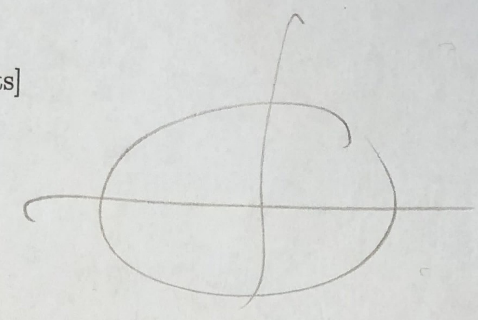
3 5. a). Evaluate

$$\lim_{x\to 0}\frac{e^x - x - 1}{\cos x - 2}.$$ [3 points]

1 -

2 b). Evaluate the integral

$$\int_1^8 \ln(x)dx.$$ [3 points]



a. $\frac{1-0-1}{1-2} = \frac{0}{-1} = 0$

$u = x$

b | $\int_1^8 \ln x \, dx$ $u = \ln x$ $du = \cancel{\frac{1}{x}}$

$v = x$ $dv = 1$

$\boxed{\left(8\ln 8 - \frac{64}{2}\right) - \left(\ln 1 - \frac{1}{2}\right)}$

$x \ln x - \int \cancel{x}$

$x \ln x - \frac{x^2}{2}\cancel{x}\Big|_1^8$

$x \ln x - \int x$

$x \ln x - \frac{x^2}{2}\Big|_1^8$