1. For each function, find dy/dx . Write your answers in the space provided .

a. $y = \ln(\cos x)$

a. $-\tan(x)$

$$y' = \frac{1}{\cos x} \cdot -\sin x = \frac{-\sin x}{\cos x} = -\tan x$$

b. $y = (3x)^{\frac{1}{x}}$

b. $\frac{1-\ln(3x)}{x^2}\left(3x^{\frac{1}{x}}\right)$

$$\ln y = \ln(3x)^{\frac{1}{x}}$$

$$\ln y = \frac{1}{x}\ln(3x) = x^{-1}\ln(3x)$$

$$\frac{1}{y}y' = \frac{1}{x}\cdot\frac{1}{3x}3 + \ln(3x)\cdot -x^{-2}$$

$$\frac{1}{y}y' = \frac{3}{3x^2} + \ln(3x)\cdot\frac{-1}{x^2} = \frac{1}{x^2} - \frac{\ln(3x)}{x^2} = \frac{1-\ln(3x)}{x^2}$$

$$y' = \frac{1-\ln(3x)}{x^2}(y)$$

$$y' = \frac{1-\ln(3x)}{x^2}\left(3x^{\frac{1}{x}}\right)$$

c. $tan^{-1}(x^2)$

c. $\frac{2x}{1-x^4}$

$$y' = \frac{1}{1-(x^2)^2}2x = \frac{1}{1-x^4}(2x) = \frac{2x}{1-x^4}$$

2. Evaluate each definite integral . Write your answer in the space provided .

a. $\int_{-1}^{1} 3x^2\sqrt{x^3+1}\ dx$

a. $\frac{4\sqrt{2}}{3}$

$u = x^3+1$

$\frac{du}{dx} = 3x^2$

$dx = \frac{1}{3x^2}du$

$\int 3x^2\sqrt{u}\cdot\frac{1}{3x^2}du = \int \sqrt{u}\,du = \int u^{\frac{1}{2}}du = \frac{2}{3}u^{\frac{3}{2}}$

$u(-1) = -1+1 = 0$

$u(1) = 1+1 = 2$

$\frac{2}{3}u^{\frac{3}{2}}\Big|_0^2 = \frac{2}{3}(2^{\frac{3}{2}}) - \frac{2}{3}(0^{\frac{3}{2}}) = \frac{2}{3}(2\cdot\sqrt{2} - 0\cdot\sqrt{0}) = \frac{2}{3}\cdot 2\sqrt{2} = \frac{4\sqrt{2}}{3}$

6/6

b. $\int_0^{\pi/2} x\cos(2x)\,dx$

b. $-\frac{1}{2}$

$u = x$ | $v = \frac{1}{2}\sin(2x)$

$du = dx$ | $dv = \cos(2x)dx$

6/6

$\int x\cos(2x)dx = \frac{1}{2}x\sin(2x) - \int\frac{1}{2}\sin(2x)dx$

$= \frac{1}{2}x\sin(2x) - \frac{1}{2}\int\sin(2x)dx$

$= \frac{1}{2}x\sin(2x) - \frac{1}{2}\cdot -\cos(2x)\cdot\frac{1}{2}$

$= \frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x)$

check: $\frac{1}{2}x\ 2\cos(2x) + \sin(2x)\frac{1}{2} + \frac{1}{4}\cdot 2\cdot -\sin(2x)$

$x\cos(2x) + \frac{1}{2}\sin(2x) - \frac{1}{2}\sin(2x)$

$x\cos(2x)$ ✓

$\frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x)\Big|_0^{\pi/2} = \frac{1}{2}\frac{\pi}{2}\sin(\pi) + \frac{1}{4}\cos(\pi) - (0 + \frac{1}{4}\cos(0))$

$= \frac{\pi}{4}(0) + \frac{1}{4}(-1) - \frac{1}{4}(1)$

$= -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}$

3. Use L'Hôpital's rule (if necessary) to evaluate each limit . If you use L'Hôpital's rule, state the type of indeterminate form you encounter .

a. $\lim\limits_{x\to\pi/2} \dfrac{1-\sin(x)}{1+\cos(2x)}$ $\quad \dfrac{1-\sin(\frac{\pi}{2})}{1+\cos(\pi)} = \dfrac{1-1}{1+-1} = \dfrac{0}{0}$ indeterminate form $\frac{0}{0}$

$$\lim_{x\to\frac{\pi}{2}} \frac{-\cos(x)}{-\sin(2x)\cdot 2} = \lim_{x\to\frac{\pi}{2}} \frac{\cos(x)}{2\sin(2x)} = \frac{\cos(\frac{\pi}{2})}{2\sin(\pi)} = \frac{0}{2(0)} = \frac{0}{0} \text{ again}$$

$$\lim_{x\to\frac{\pi}{2}} \frac{-\sin(x)}{2\cdot 2\cos(2x)} = \lim_{x\to\frac{\pi}{2}} \frac{-\sin(x)}{4\cos(2x)} = \frac{-\sin(\frac{\pi}{2})}{4\cos(\pi)} = \frac{-1}{4(-1)} = \frac{-1}{-4} = \frac{1}{4} \checkmark$$

b. $\lim\limits_{x\to\infty} (\ln x^{\frac{1}{x}})$ $= \lim\limits_{x\to\infty} \frac{1}{x}\ln x$

$$e^{\lim\limits_{x\to\infty} \frac{1}{x}\ln(\ln x)}$$

1/4

2/4

c. $\lim\limits_{x\to 0^+} (\frac{1}{x} - \frac{1}{\sqrt{x}})$

$$\lim_{x\to 0^+}\left(\frac{\sqrt{x}}{\sqrt{x}\,x} - \frac{x}{\sqrt{x}\,x}\right) = \lim_{x\to 0^+}\left(\frac{\sqrt{x}-x}{x\sqrt{x}}\right) = \frac{0-0}{0} = \frac{0}{0} \text{ indeterminate form } \frac{0}{0}$$

$$\lim_{x\to 0^+} \frac{\frac{1}{2}x^{-\frac{1}{2}}-1}{x\cdot\frac{1}{2}x^{-\frac{1}{2}}+\sqrt{x}} = \lim_{x\to 0^+} \frac{\frac{1}{2\sqrt{x}}-1}{\frac{x}{2\sqrt{x}}+\sqrt{x}} \checkmark = \lim_{x\to 0^+} \frac{\frac{1-2\sqrt{x}}{2\sqrt{x}}}{\frac{x}{2\sqrt{x}}+\frac{2x}{2\sqrt{x}}} = \lim_{x\to 0^+} \frac{\frac{1-2\sqrt{x}}{2\sqrt{x}}}{\frac{3x}{2\sqrt{x}}} \Rightarrow \frac{1-2\sqrt{x}}{3x} \to \frac{1}{0}\ \infty$$

$$\lim_{x\to 0^+} \left(\frac{(1-2\sqrt{x})2\sqrt{x}}{(2\sqrt{x})3x}\right) = \lim_{x\to 0^+} \frac{2\sqrt{x}-4x}{6x\sqrt{x}} = \frac{0-0}{0}$$

$$\lim_{x\to 0^+} \frac{2\cdot\frac{1}{2}x^{-\frac{1}{2}}-4}{6x\cdot\frac{1}{2}x^{-\frac{1}{2}}+6\sqrt{x}} = \lim_{x\to 0^+} \frac{\frac{1}{\sqrt{x}}-4}{\frac{6x}{2\sqrt{x}}+6\sqrt{x}} = \lim_{x\to 0^+} \frac{\frac{1}{\sqrt{x}}-4}{\frac{3x}{\sqrt{x}}+6\sqrt{x}}$$

4. Sketch the graph of $y = e^{-x^2} + 3$. Find the critical point(s) , the inflection point(s) , any asymptotes . Label the critical / inflection points on your graph .

critical points:

$y' = e^{-x^2} \cdot -2x$

$y' = -2xe^{-x^2}$

$y' = 0$

$-2xe^{-x^2} = 0$

$-2x = 0$ when $x = 0$

$e^{-x^2} = 0$ never

$y'(1) = -2(1)e^{-1} = -2 \cdot \frac{1}{e} = \frac{-2}{e} < 0$

$y'(-1) = -2(-1)e^{-1} = 2e^{-1} = \frac{2}{e} > 0$

$y(0) = -e^0 + 3 = 1 + 3 = 4$

y': $+$ | 0 | $-$ ✓

y: $\nearrow$ | 0 | $\searrow$

y has a local maximum at $(0, 4)$.

Inflection points:

$y'' = -2x \cdot -2x \cdot e^{-x^2} + e^{-x^2} \cdot -2$

$y'' = 4x^2 e^{-x^2} - 2e^{-x^2}$

$y'' = 2e^{-x^2}(2x^2 - 1)$

$2e^{-x^2} = 0$

$e^{-x^2} = 0$ never

$2x^2 - 1 = 0$

$2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = \pm \frac{1}{4}$ $\quad x = \pm\sqrt{1/2}$

$y''(-1) = 2e^{-1}(2-1) = \frac{2}{e}(1) > 0$

$y''(0) = 2e^0(0-1) = 2(-1) = -2 < 0$

$y''(1) = 2e^{-1}(2-1) = \frac{2}{e}(1) > 0$

y'': $+$ | $-\frac{1}{4}$ | $-$ | $\frac{1}{4}$ | $+$ ✓

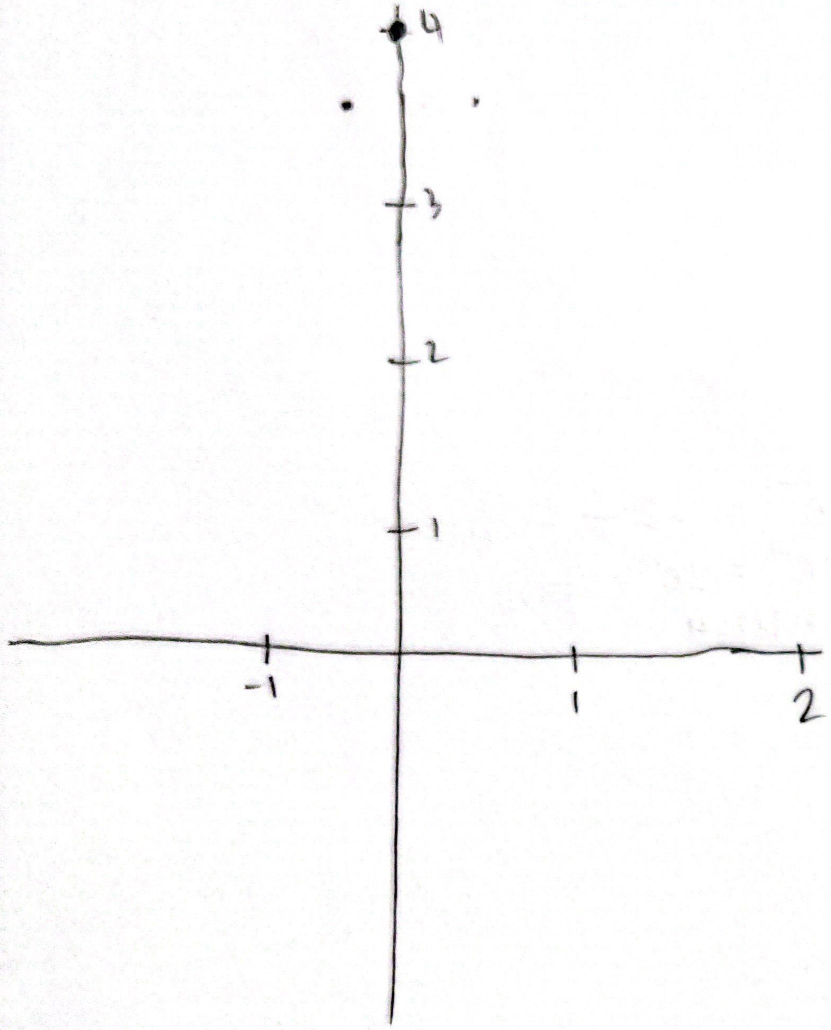
y: $\cup$ | $-\frac{1}{4}$ | $\cap$ | $\frac{1}{4}$ | $\cup$

Use the next page if you need more room for calculations.

$y(-\frac{1}{4}) = e^{-\frac{1}{2}} + 3 = \frac{1}{\sqrt{e}} + 3 \approx 3.61$

$y(\frac{1}{4}) = e^{-\frac{1}{2}} + 3 = \frac{1}{\sqrt{e}} + 3 \approx 3.61$

y changes concavity from up to down at $(-\frac{1}{4}, \frac{1}{\sqrt{e}} + 3)$ and from down to up at $(\frac{1}{4}, 3.61$


4
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