Problem 1. *1pts.*

Find the derivative of $f(x) = \frac{e^{x^2}}{x}$ at $x = 1$.

[✓ (a) e]

(b) $2e$

(c) $\frac{e}{2}$

(d) e^2

e^{x^2}

1.

$e^{\ln}$

$\ln e = 1$

$\ln(1) = 0$

$\frac{f'(x)}{f(x)} = \ln e^{x^2} - \ln x$

$(x^2 \ln e - \ln x) \frac{e^1}{1}$

$x^2 \frac{1}{e} + \ln e 2x - \frac{1}{x}$

$\frac{1}{\ }$ $\frac{1}{e} +$

$\left(\frac{1}{e} + 2\ln e - 1\right) e$

$1 + 2e - e$

$e^{x^2} \cdot 2x$

x

$\left(\ln(e^{x^2}) - \ln(x)\right)$

$x^2 \ln e - \ln x$

$\left(\frac{x^2}{e} + (\ln e)^{2x} - \frac{1}{x}\right) \frac{e^{x^2}}{x}$

$\left(\frac{1}{e} + e^2 - 1\right) \frac{e^1}{1}$

$+ e^3 - e$

$\frac{x \frac{d}{dx} e^{x^2} - e^{x^2}}{x^2}$

$\frac{1(e^{x^2}) \cdot 2 - e}{1}$

$2e^1 \cdot 2 - e$

$2e - e = \boxed{e}$

Problem 2. *1pts.*

Find the inverse of $f(x) = \frac{1}{7x-3}$ at $x = 1$.

(a) $\frac{1}{7}$

(b) $\frac{2}{7}$

[✓ (c) $\frac{3}{7}$]

[✓ (d) $\frac{4}{7}$]

~~g~~ f^{-}

$$y = \frac{1}{7x-3}$$

$$x = \frac{1}{7y-3}$$

$$\underset{+3}{7y-3} = \frac{1}{x} + 3$$

$$\frac{7y}{7} = \frac{\frac{1}{x}+3}{7}$$

$$y = \frac{3}{7}$$

Problem 3. *1pts.*

Find the derivative of $f(x) = x^{3x}$ at $x = \frac{1}{3}$.

(a) $1 + \ln(3)$

[✓ (b) $1 - \ln(3)$]

(c) $3 + 3\ln(3)$

(d) $3 - 3\ln(3)$

$x = e^{\ln x}$

$e^{\ln(x)^{3x}}$

$e^{3x(\ln x)}$

$\left(\frac{3\cancel{x}}{\cancel{x}} + 3\ln\right)x^{3x}$

$\left(3 + 3\ln\frac{1}{3}\right)\left(\frac{1}{3}^{1}\right)$

$\ln\frac{1}{3} = (\ln 1) - \ln(3)$

$1 -$

$\ln\left(\frac{1}{3}\right) + 1$

$\cancel{\ln(1)} - \ln(3) + 1$

$1 - \ln 3$

Problem 4. *2pts.*

90°

Find the limit of $f(x) = \sec(x) - \tan(x)$ at $x = \frac{\pi}{2}$.

(a) 1

[✓ (b) 0] ✓

(c) $+\infty$

(d) $-\infty$

$\sec = \frac{1}{\cos x}$

cos at 0 = 1

sin at 1 = 0

$\frac{1}{\cos x} - \frac{\sin x}{\cos x}$

~~(π,0~~ (1,0)

($\pi/2$ (0,1)

$\frac{1-\sin x}{\cos x}$ $\frac{0}{0}$

$\frac{\pi}{2}$

$\frac{-\cos x}{\sin x}$ $\frac{0}{1}$

Problem 5. *1pts.*

Find the integral of $f(x) = \frac{1}{\sqrt{1-16x^2}}$ between 0 and $\frac{1}{4}$.

- (a) π
- (b) $\frac{\pi}{2}$ ✓
- (c) $\frac{\pi}{4}$
- [✓ (d) $\frac{\pi}{8}$]

$$\int \frac{1}{\sqrt{}}$$

u=

(4x) = u

u = 4x

du = 4dx

$\frac{du}{4} = dx$

$$\frac{1}{4}\int \frac{1}{\sqrt{1-u^2}}$$

$$\frac{1}{4}\arcsin(4x)\Big|_0^{1/4}$$

$$\frac{1}{4}\left(\underset{\frac{\pi}{2}}{\arcsin(1)} - \underset{0}{\arcsin(0)}\right]$$

$\arcsin \cancel{x} = \cancel{\theta}$

$\sin\theta = x$

at 0 (0,1) — sin cosine

$\frac{\pi}{2}$ (1,0) — sin cosine

Problem 6. *1pts.*

Find the integral of $f(x) = e^{-x}\sinh(x)$ between 0 and $\frac{1}{2}$.

[✓ (a) $\frac{1}{4e}$]

(b) $\frac{1}{4\cosh(1)}$

[✓ (c) $\frac{1}{4\sinh(1)}$]

(d) $\frac{1}{4\tanh(1)}$

$\sinh x = \frac{e^{x}-e^{-x}}{2}$

$\int e^{-x}\sinh(x)$

e

$\int \cancel{e^{-x}}-\cancel{e^{x}}$

$f(x)=\int e^{-x}\sinh(x)$

$\frac{-e^{-x}\sinh x - \cosh x}{4}$

$\int \frac{e^{-x}(e^{x}-e^{-x})}{2}$

$u=\sinh x \quad du=\cosh x$

$v'=e^{-x} \quad v=-e^{-x}$

$\frac{1}{2}\int e^{-2x}-e^{2x}$

$-e^{-x}\sinh x - \int \cosh x e^{-x}$

$u=2x$

$du=\frac{1}{2}$

$u=\cosh x \quad du=+\sinh(x)$

$v'=-e^{-x} \quad v=+e^{-x}$

$\frac{1}{2}$

$\frac{1}{4}\int e^{-u}-e^{u}$

$-e^{-x}\sinh x - (\cosh x(+e^{-x}) + \int \sinh(x)(+e^{-x}))$

$\frac{1}{4}\cancel{\int}-e^{-u}-e^{u}$

$\cosh x = \frac{e^{x}+e^{-x}}{2}$

$e^{-x}(e^{x}-e^{-x})$

$\frac{e^{-2x}-e^{2x}}{2}$

$u=2x$

$-\frac{1}{4}(e^{-u}+e^{u})$

$\frac{1}{4}(\int -1(\cosh x)$

$\cosh x$

$\frac{1}{4}\int e^{-u}-e^{u}$

$=\int \cosh x$

$-\frac{1}{4}\sinh x$

$\frac{1}{4}\int -1(e^{u}-e^{-u})$

$\sinh x$

$\sinh x - \int \cosh x(-x)$

$\cosh x(-x)$

$u=\sinh x \quad du=\cosh x$

$v'=-1 \quad v=-x$

Problem 7. *2pts.*

Find the integral of $f(x) = x\ln(x)$ between 1 and 2.

(a) $\ln(2) + \frac{3}{4}$

(b) $\ln(4) + \frac{3}{4}$

~~(c) $\ln(2) - \frac{3}{4}$~~

[✓ (d) $\ln(4) - \frac{3}{4}$]

$f(x) = \int x\ln(x)$

~~$u = x \quad du = 1$~~

$u = \ln x \quad du = \frac{1}{x}$

$v' = x \quad v = \frac{x^2}{2}$

$\ln x\left(\frac{x^2}{2}\right) - \int \frac{x^2}{2} \cdot \frac{1}{x}$

$\ln x\left(\frac{x^2}{2}\right) - \frac{1}{2}\int x^3$

$\ln x\left(\frac{x^2}{2}\right) - \frac{1}{2}\frac{x^4}{4}\Big|_1^2$ $\quad 2^4 \quad 2 \cdot 2$

$\left(\ln 2(2) - \frac{16}{8}\right) - \left(\cancel{\ln(1)\left(\frac{x^2}{2}\right)} - \frac{1}{8}\right)$

$2\ln 2 - \frac{15}{8}$

$\ln(x)\left(\frac{x^2}{2}\right) - \frac{1}{2}\frac{x^2}{2}$

$\ln(2)\left(\frac{4}{2}\right) - \frac{1}{2}\left(\frac{4}{2}\right) - \left(\ln(1)\left(\frac{1}{2}\right) - \frac{1}{2}\left(\frac{1}{2}\right)\right)$

$2\ln(2) - 1$

$-\frac{3}{4}$

Problem 8. *1pts.*

Find the integral of $f(x) = \frac{3x^2-4x+5}{x^3-x^2+x-1}$ between 2 and 3.

[✓ (a)] $\frac{1}{2}\left(\ln(32) + 6\arctan(2) - 6\arctan(3)\right)$

(b) $\frac{1}{2}\left(\ln(32) + 6\arctan(3) - 6\arctan(2)\right)$

X

[✓ (c)] $\frac{1}{2}\left(\ln(16) + 6\arctan(2) - 6\arctan(3)\right)$

(d) $\frac{1}{2}\left(\ln(16) + 6\arctan(3) - 6\arctan(2)\right)$

x^2+1

$x-1 \;\overline{)\; x^3-x^2+x-1}$

$-(x^3+x^2)$

$0 \quad x$

$-x$

$\boxed{A=2}$

$(x-1)(x^2+1) = \frac{A}{x-1} + \frac{Bx+C}{(x^2+1)}$

$3x^2-4x+5 = A(x^2+1) + (Bx+C)(x-1)$

$Bx+C$

$x \quad Bx^2 \quad Cx$

$-1 \quad -Bx \quad -C$

$3-4+5$

$-1+5=4$

$x=1$

$4 = A(2)$

$= 2x^2+2+Bx^2+Cx-Bx-C$

$\boxed{B=1}$ $\quad \boxed{C=\overset{-3}{\cancel{4}}}$

$4=(Cx-Bx$

$\frac{x+5}{x^2+1}$

$\boxed{2\ln|x-1|}$ $\quad \int\frac{x}{x^2+1} \rightarrow \int\frac{5}{x^2+1}$

arc tangent?

$3=2+B$

$B=4$

$u=x^2+1$

$du=2x$

$2\ln|x-1| + \frac{1}{2}\ln|x^2+1| \mp 5(\arctan(x))$

$\int\frac{x}{u}\cdot\frac{1}{2\cancel{x}}$ $\quad 3-1=2$

$2\ln|2| + \frac{1}{2}\ln|10| \mp 5(\arctan 3)$

$-\cancel{2\ln|1|} + \frac{1}{2}\ln|2| - 3(\arctan 2)$