20W-MATH31B-1 Midterm 1

TOTAL POINTS

27.5 / 40

QUESTION 1

1 Problem 1(a) 2 / 4

- **- 1 pts** Minor Mistatkes
- ✓ **- 2 pts** **Serious Mistakes or Answer Not Simplified**
- **- 3 pts** Incorrect or Only Show Some Efforts
- **- 4 pts** Blank or Completely Wrong
- **- 0 pts** All Correct

QUESTION 2

2 Problem 1(b) 4 / 4

- **- 1 pts** Minor Computation Mistakes
- **- 2 pts** Serious Mistakes
- **- 3 pts** Incorrect but Show Some Efforts
- **- 4 pts** Blank or Completely Incorrect
- ✓ **- 0 pts** **All Correct**

QUESTION 3

3 Problem 1(c) 3 / 3

- **- 1 pts** Not Adding Constant C
- **- 1 pts** Minor Computation Mistakes
- **- 2 pts** Serious Mistakes or Just Tried Substitution
- **- 3 pts** Blank or Completely Incorrect
- ✓ **- 0 pts** **All Correct**

QUESTION 4

4 Problem 2(a) 3 / 4

- ✓ **+ 0.5 pts** **Write lim f(x)/g(x) [or lim g(x)/f(x)]**
- ✓ **+ 0.5 pts** **Find that lim = infty/infty**
- ✓ **+ 0.5 pts** **Apply L'Hopital's rule**
- **+ 0.5 pts** Correct computation of f'(x)/g'(x)
- ✓ **+ 0.5 pts** **lim f'(x)/g'(x) = infty/infty + Apply L'Hopital's rule again**
- **+ 0.5 pts** Correct computation of ratio of derivatives
- ✓ **+ 0.5 pts** **lim = 0 [or infty]**
- ✓ **+ 0.5 pts** **Deduce that f(x) << g(x)**
- **+ 0 pts** Incorrect or not attempted

QUESTION 5

5 Problem 2(b) 3 / 4

- ✓ **+ 0.5 pts** **Write lim f(x)/g(x)**
- **+ 2 pts** Take exponents out of the natural logarithm
- ✓ **+ 1 pts** **Compute limit**
- ✓ **+ 0.5 pts** **Deduce that neither holds**
- ✓ **+ 0.25 pts** **infty/infty**
- ✓ **+ 0.25 pts** **Apply L'Hopital's rule**
- **+ 1.5 pts** Correct derivatives
- ✓ **+ 0.5 pts** **Derivatives w/ 1 mistake**
- **+ 0 pts** Derivatives w/ 2 mistakes
- **+ 0 pts** Incorrect or not attempted

QUESTION 6

6 Problem 3(a) 5 / 5

- ✓ **- 0 pts** **Correct**
- **- 2 pts** Took the derivative but incorrect / missing reasoning
- **- 4 pts** Took the derivative but very incorrect / missing reasoning
- **- 1 pts** Took the derivative but slightly incorrect / missing reasoning
- **- 5 pts** Incorrect
- **- 3 pts** Vague reasoning to show the function is decreasing
- **- 4 pts** Very vague / incorrect reasoning
- **- 1 pts** Minor algebra or calculus mistake

QUESTION 7

7 Problem 3(b) 2 / 3

- **- 0 pts** Correct
- **- 2 pts** Correct range, incorrect domain, no explanation
- ✓ **- 1 pts** **Correct range, incorrect domain, but some**

explanation or indication of understanding that Domain(g) = Range(f)

- 3 pts Incorrect

QUESTION 8

8 Problem 3(c) 0 / 3

- 0 pts Correct

- 2 pts Considered the correct equation but either gave incorrect solution, specified both solutions, or failed to give any solution

✓ **- 3 pts** **Incorrect**

QUESTION 9

9 Problem 4(a) 2.5 / 3

+ 3 pts Correct

+ 2 pts Minor errors or missing details.

+ 1 pts Major errors, with progress.

+ 1 pts Right idea with no progress on limit computations

+ 0 pts No credit

+ 2.5 Point adjustment

You need to distinguish between your 4's and 7's. It's not clear what you are dividing by in your "algebra" and that absolutely matters.

QUESTION 10

10 Problem 4(b) 2 / 3

+ 3 pts Correct

+ 1 pts Major error with L'Hopital.

+ 2 pts Messed up limit after L'Hopital.

+ 2 pts Minor error with L'Hopital.

+ 0 pts No credit

+ 2 Point adjustment

bounds are wrong.

QUESTION 11

11 Problem 4(c) 1 / 4

+ 4 pts Correct

+ 2 pts Major errors, made progress

+ 0 pts No credit

+ 3 pts Minor errors, made progress

✓ **+ 1 pts** **Rewrote as log/exponential but made no progress.**

Math 31B
Integration and Infinite Series

Midterm 1

Instructions: You have 50 minutes to complete this exam. There are 4 questions, worth a total of 40 points. This test is closed book and closed notes. No calculator is allowed.

For full credit show all of your work legibly and justify your answers.

Please write your solutions in the space below the questions. If you run out of space, please *do not write on the back of the page*, but continue on the extra sheets attached at the end of the booklet. Please INDICATE if you continue on the extra sheets.

Do not forget to write your name, section and UID legibly in the space below.

Name: _____

Student ID number: _____

Section (circle one):

	Tuesday	Thursday
Weiyi Liu	1A	1B
Timothy mits	1C	1D
Joseph Breen	1E	1F

Question	Points	Score
1	11	
2	8	
3	11	
4	10	
Total:	40	

FORMULAE

Trigonometric identities

$$\cos^2(x) + \sin^2(x) = 1$$

$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$
$$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$

$$\frac{d}{dx}\sin(x) = \cos(x)$$
$$\frac{d}{dx}\cos(x) = -\sin(x)$$

Problem 1.

(a) [4pts.] Compute the derivative of $f(x) = \dfrac{x \cdot (\sqrt[4]{x+1})^3}{(\sqrt{x-1})^5 \cdot (x^3+3)}$.

$$f(x) = \frac{x \cdot (x+1)^{\frac{3}{4}}}{(x-1)^{\frac{5}{2}}(x^3+3)}$$

$$f'(x) = \frac{\left((x)'(x+1)^{3/4} + (x)(x+1)^{3/4}\right)(x-1)^{5/2}(x^3+3) - x(x+1)^{3/4}\left(\frac{5}{2}(x-1)^{3/2}(x^3+3) + (x-1)^{5/2}3x^2\right)}{\left((x-1)^{5/2}(x^3+3)\right)^2}$$

$$f'(x) = \frac{\left((x+1)^{3/4} + \frac{3}{4}x(x+1)^{-1/4}\right)(x-1)^{5/2}(x^3+3) - x(x+1)^{3/4}\left(\frac{5}{2}(x-1)^{3/2}(x^3+3) + (x-1)^{5/2}3x^2\right)}{\left((x-1)^{5/2}(x^3+3)\right)^2}$$

(b) [4pts.] Compute the derivative of $g(x) = (\sin x)^{\log_2 x}$.

$$\ln(g(x)) = \ln((\sin x)^{\log_2 x})$$

$$\frac{d}{dx}\ln(g(x)) = \log_2 x \ln(\sin x) \quad \text{(product rule)}$$

$$\frac{g'(x)}{g(x)} = \frac{1}{x \ln 2}\ln(\sin x) + \log_2 x \frac{\cos x}{\sin x}$$

$$g'(x) = ((\sin x)^{\log_2 x})\left(\frac{\ln(\sin x)}{x \ln 2} + \frac{\log_2 x \cos x}{\sin x}\right)$$

(c) [3pts.] Evaluate $\int \left(\frac{1}{7}\right)^{-7x+6} dx$

$$\frac{d}{dx} b^x = b^x \ln b \cdot (x)'$$

$$u = \left(\frac{1}{7}\right)^{-7x+6}$$

$$du = \left(\frac{1}{7}\right)^{-7x+6} \ln\left(\frac{1}{7}\right) \cdot -7\, dx$$

$$\frac{du}{-7\ln\left(\frac{1}{7}\right)} = \left(\frac{1}{7}\right)^{-7x+6} dx$$

$$\boxed{\frac{\left(\frac{1}{7}\right)^{-7x+6}}{-7\ln\left(\frac{1}{7}\right)} + C}$$

Problem 2.

For each of the following pairs of functions $f(x)$ and $g(x)$, say whether $f(x) \gg g(x)$, $f(x) \ll g(x)$, or neither of them holds.

(a) [4pts.] $f(x) = (\ln x)^2$ and $g(x) = 2^{\ln x}$

$$\lim_{x\to\infty} \frac{(\ln x)^2}{2^{\ln x}} = \frac{\infty}{\infty}$$

L'Hopital's

$$\lim_{x\to\infty} \frac{2\ln x \cdot \frac{1}{x}}{\ln(\ln x)\cdot 2^{\ln x}\cdot\frac{1}{x}} = \frac{\infty}{\infty}$$

L'Hopital's

$$\lim_{x\to\infty} \frac{2\cdot\frac{1}{x}}{\ln(\ln x)\cdot\ln(\ln x)\cdot 2^{\ln x}\cdot\frac{1}{x}} = \frac{2}{\infty} = 0.$$

Since the

$$\lim_{x\to\infty} \frac{(\ln x)^2}{2^{\ln x}} = 0, \quad 2^{\ln x} \text{ is growing faster than } (\ln x)^2.$$

Therefore,

$$\boxed{f(x) \ll g(x)}$$

$f(x)$ $g(x)$

(b) [4pts.] $\ln((x\ln x)^2)$ and $\ln(\sqrt[3]{x\ln x})$

$$\lim_{x\to\infty} \frac{\ln((x\ln x)^2)}{\ln(\sqrt[3]{x\ln x})} = \frac{\infty}{\infty}$$

L'Hopital's

$$\lim_{x\to\infty} \frac{2(x\ln x)\cdot\left((x)'(\ln x)+(x)(\ln x)'\right)\cdot\frac{1}{(x\ln x)^2}}{\frac{1}{3}(x\ln x)^{-2/3}\cdot\left((x)'(\ln x)+(x)(\ln x)'\right)} = \frac{2(\ln x+1)}{x\ln x} \cdot \frac{3(x\ln x)^{2/3}}{(\ln x+1)}$$

$$\lim_{x\to\infty} \frac{\frac{2}{x\ln x}}{\frac{1}{3(x\ln x)^{2/3}}} = \lim_{x\to\infty} \frac{\frac{2}{(x\ln x)^{1/3}}}{1/3} = \frac{\frac{2}{\infty}}{1/3} = 0$$

Since $\lim_{x\to\infty} \frac{\ln((x\ln x)^2)}{\ln(\sqrt[3]{x\ln x})} = 0$, $g(x)$ is growing faster than $f(x)$.

$$f(x) << g(x)$$

Problem 3.

Let $f(x)$ be the function with domain $D = (-\infty, 0]$ defined by $f(x) = x^{10} + x^2 + 1$.

(a) [5pts.] Show that $f(x)$ is one-to-one (on the domain D).

one-to-one if increasing or decreasing

$f'(x) = 10x^9 + 2x$ Derivative is negative on interval $(-\infty, 0]$

$\lim_{x \to -\infty} 10x^9 + 2x = -\infty$ Therefore, $f(x)$ is decreasing on $(-\infty, 0]$

$f'(0) = 0 + 0 = 0$ and is one-to-one.

(b) [3pts.] If $g(x)$ denotes the inverse of $f(x)$, what are the domain and the range of $g(x)$?

Range is $f(x)$'s domain

$\boxed{g(x) \text{ Range} = (-\infty, 0]}$

Domain is $f(x)$'s range

$\lim_{x \to -\infty} x^{10} + x^2 + 1 = -\infty$

$f(0) = 0 + 0 + 1 = 1$

$\boxed{g(x) \text{ Domain} = (-\infty, 1]}$

(c) [3pts.] Compute $g(3)$.

Cannot compute as it is not on the domain of $g(x)$. Domain is $(-\infty, 1]$

Problem 4.

Compute the following limits.

(a) [3pts.] $\lim_{n\to+\infty} \dfrac{3^n + 4^n \cdot \sin n}{7^n}$

Hint: recall that $-1 \le \sin x \le 1$ for all x.

Limit does not work with sin(n)

Squeeze thm

$-1 \le \sin n \le 1$

$-4^n \le 4^n \sin n \le 4^n$

$3^n - 4^n \le 4^n \sin n + 3^n \le 4^n + 3^n$

$\dfrac{3^n - 4^n}{7^n} \le \dfrac{3^n + 4^n \sin n}{7^n} \le \dfrac{4^n + 3^n}{7^n}$

$\lim_{n\to\infty} \dfrac{3^n - 4^n}{7^n} \le \lim_{n\to\infty} \dfrac{3^n + 4^n \sin n}{7^n} \le$

$\lim_{n\to\infty} \dfrac{3^n - 4^n}{7^n} = \dfrac{\infty}{\infty}$

Algebra

$\lim_{n\to\infty} \dfrac{\frac{3^n}{4} - \frac{4^n}{4}}{\frac{7^n}{4}} = 0$

$\lim_{n\to\infty} \dfrac{4^n + 3^n}{7^n} = \dfrac{\infty}{\infty}$

$\lim_{n\to\infty} \dfrac{\frac{4^n}{7} + \frac{3^n}{7}}{\frac{7^n}{7}} = 0$

$0 \le \dfrac{3^n + 4^n \sin n}{7^n} \le 0$

Since the squeeze thm states that $0 \le \dfrac{3^n + 4^n \sin n}{7^n} \le 0$,

$\lim_{n\to\infty} \dfrac{3^n + 4^n \cdot \sin n}{7^n} = 0$

(b) [3pts.] $\lim_{x\to 0} \dfrac{x^2}{3^x - \cos x}$

Limit does not work with cos(x)

$-1 \le \cos x \le 1$

$-1 \le -\cos x \le 1$

$3^x - 1 \le 3^x - \cos x \le 1 + 3^x$

$\dfrac{x^2}{3^x - 1} \le \dfrac{x^2}{3^x - \cos x} \le \dfrac{x^2}{1 + 3^x}$

ind. $\frac{0}{0}$ $\lim_{x\to 0} \dfrac{x^2}{3^x - 1} \le \dfrac{x^2}{3^x - \cos x} \le \lim_{x\to 0} \dfrac{x^2}{1 - 3^x}$ ind. $\frac{0}{0}$

L'H

$\lim_{x\to 0} \dfrac{2x}{\ln 3\, 3^x}$

L'H

$\lim_{x\to 0} \dfrac{2x}{1 \cdot 3^2 x}$

$0 \le \dfrac{x^2}{3^x - \cos x} \le 0$

$\lim = 0$

(c) [4pts.] $\lim_{x\to+\infty}\left(1+\frac{\ln 3}{x}\right)^x$

$\lim_{x\to\infty}\left(1+\frac{\ln 3}{x}\right)^x = 1^\infty$ (indeterminate)

$\lim_{x\to+\infty} e^{\ln\left(1+\frac{\ln 3}{x}\right)^x}$

$\lim_{x\to\infty} e^{x\ln\left(1+\frac{\ln 3}{x}\right)}$

$e^{\lim_{x\to\infty} x\ln\left(1+\frac{\ln 3}{x}\right)}$

$e^{\infty\cdot\ln(1)}$

$\lim_{x\to\infty}\left(1+\frac{\ln 3}{x}\right)^x = \infty$

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