Problem 1. Find the derivative of $f(x)$ if

(a) (1 point) $f(x) = xe^{3x}$

$\cancel{u}\ \cancel{dv}$ $\cancel{du}\ \cancel{v}$

$\cancel{u=x}\ \cancel{dv=e^{3x}}$ $\cancel{u=3x}$ $\cancel{\frac{1}{3}du=dx}$

$\cancel{du=1}\ \cancel{v=\frac{e^{3x}}{3}}\ \cancel{e^{u}}$

$1\cdot e^{3x}+3x\cdot e^{3x}$

$= e^{3x}+3xe^{3x}$

(b) (2 points) $f(x) = x^{x^2}$

$y = x^{x^2}$

$e^{\ln(x^{x^2})}$

$e^{x^2\ln(x)}$

$= e^{x^2\ln(x)}\cdot(2x\ln x + x^2\cdot\frac{1}{x}) = (e^{x^2\ln(x)}\cdot(2x\ln x + x)) =$

$\ln y = x^2\ln x$

$\frac{y'}{y} = 2x\ln x + \cancel{x^2/x}$

$= (2x\ln x + x)\cdot x^{x^2}$

$x^{x^2}\cdot\ldots$ "

(c) (2 points) $f(x) = \cos^{-1}(\ln x)$ (here $\cos^{-1}$ is an inverse of $\cos$, i.e. arccos)

$\frac{-1}{\sqrt{1-x^2}}$

$\frac{-1}{\sqrt{1-(\ln x)^2}}\cdot\frac{1}{x}$

$= \frac{-1}{x\sqrt{1-(\ln(x))^2}}$

Problem 2. Find the following limits

(a) (1 point) $\lim_{x\to 1}\frac{x^2-1}{x^3-1}$ $\frac{0}{0}$

$\lim_{x\to 1}\frac{2x}{3x^2} = \frac{2}{3}$

(b) (2 points) $\lim_{x\to\infty}\frac{x^3}{e^x}$ $\frac{\infty}{\infty}$

$\frac{3x^2}{e^x}$ $\frac{6x}{e^x}$ $\frac{6}{e^x} = \frac{6}{\infty} = 0$

(c) (2 points) $\lim_{x\to 0}(1+x)^{1/x}$

1^∞

$y = (1+x)^{1/x}$

$\ln y = \ln((1+x)^{1/x})$

$\lim_{x\to 0}\frac{\ln(1+x)}{x}$ ← $\frac{0}{0}$ form

$\lim_{x\to 0}\frac{1/1+x}{1} = \frac{1/1}{1} = 1 = \lim_{x\to 0}\ln y$

$\lim_{x\to 0} y = e^{\lim_{x\to 0}\ln(y)}$

$= e^{\lim_{x\to 0}\ln(y)}$

$= e^1 = e$

Problem 3. Find the following integrals

(a) (1 point) $\int \left(e^{x\sqrt{2}} + \frac{1}{x}\right) dx$

$u = x\sqrt{2}$
$du = \sqrt{2}\,dx$
$\frac{1}{\sqrt{2}}du = dx$

$= \int e^{x\sqrt{2}}dx + \int \frac{1}{x}\,dx \longrightarrow \ln|x| + C$

$= \frac{1}{\sqrt{2}}\int e^u\,du$

$= \frac{e^u}{\sqrt{2}} = \frac{e^{x\sqrt{2}}}{\sqrt{2}} + \ln|x| + C$

(b) (2 points) $\int \sin^3 x\,dx$

$\sin^2 x + \cos^2 x = 1$
$\sin^2 x = 1 - \cos^2 x$

$u = \cancel{\sin x}\ \cos x$
$du = -\sin x\,dx$
$\frac{1}{-\sin x}du = dx$

$\int (\sin x)\cdot(1-\cos^2 x)dx$

$\int \cancel{\sin x}(1-u^2)\cdot\frac{1}{\cancel{\sin x}}\,du$

$= \int -1 + u^2\,du$

$= \int -1\,du + \int u^2\,du$

$\int -1 + \int u^2$

$= -u + \frac{u^3}{3} + C$

$= -\cos x + \frac{\cos^3 x}{3} + C$

(c) (2 points) $\int x^2 \cosh x\,dx$ (here cosh is a hyperbolic cos)

u	dv
x^2	$\cosh x$
du	v
$2x$	$\sinh x$

$\int x^2 \cosh x = x^2 \sinh x - \int 2x \sinh x$

$\int 2x \sinh x = 2x\cosh x - \int 2\cosh x$

u	dv
$2x$	$\sinh x$
du	v
2	$\cosh x$

$\int 2\cosh x = 2\sinh x$

$\int x^2 \cosh x = x^2 \sinh x - (2x\cosh x - 2\sinh x)$

$= x^2\sinh x - 2x\cosh x + 2\sinh x + C$

Problem 4. (5 points) Prove that

$$\log_2 x = \frac{\ln x}{\ln 2}$$

We know: $(\log_b x)' = \frac{1}{x \ln b}$

$= \frac{1}{x}\cdot\frac{1}{\ln b}$

$f(x) = \log_b(x)$
$h(x) = (\log_b(x))'$
$\int h(x) = f(x) + C$
$\int h(x) = \frac{1}{\ln b}\int\frac{1}{x}$

$f(x) = \ln(x)\cdot\frac{1}{\ln(b)}$

$\log_b(x) = \frac{\ln x}{\ln(b)}$

so when $b = 2$

$\log_2 x = \frac{\ln x}{\ln 2}$

$\ln 2^n = \ln x$
$n\ln 2 = \ln x$
$n = \frac{\ln x}{\ln 2}$

$\log_b x = \frac{\log_a x}{\log_a b}$

$a = e,\ b = 2$

$\log_2 x = \frac{\log_e x}{\log_e 2} = \frac{\ln x}{\ln 2}$

My Answer for Problem 4:

5

n = some answer

$\log_2 x = n$

$2^n = x$ — def. of log.

$\ln(2^n) = \ln(x)$ — ln both sides

$n\ln(2) = \ln(x)$ — prop. of log.

$n = \frac{\ln(x)}{\ln(2)}$ — divide

$\log_2 x = \frac{\ln(x)}{\ln(2)}$ — substitute

∗ proved.