Final Exam
Math 31B, Winter 2021

Name:	**UID:**

Honor Statement
I assert, on my honor, that I have not received assistance of any kind from any other people, including posting exam questions on online forums while working on this Final Exam. I have only used non-human resources, for example Internet, calculators, textbook, notes, and lecture videos, during the period of this evaluation.

Signature: ______________________

Directions—Please read carefully!

- You have a 24-hour window

  Mar 13 (Sat) 8am PST – Mar 14 (Sun) 9am PDT

  to complete the exam, but the exam is designed to be able to be finished in 3 hours.

- You are allowed to use any non-human resources including internet, calculators, textbook, notes, lecture videos, etc. **You are NOT allowed to seek help from other people, including posting exam questions on online forums.**

- In order to receive full credit, you must **show your work or explain your reasoning**; your final answer is less important than the reasoning you used to reach it. Correct answers without work will receive little or no credit. Please write neatly. **Illegible answers will be assumed to be incorrect.** Circle or box your final answer when relevant.

- On Mar 13 (Sat) 8–9pm PST, I will be on Zoom for any questions about statements of exam problems. I will also be monitoring emails and Piazza notifications more closely, during normal awake time at PST.

- The exam is on Gradescope. Please either
  - Write your answers on the pdf file of the exam, then submit onto Gradescope,
  - Print the exam and write your answers on the exam paper, then scan and submit onto Gradescope, or
  - Use blank sheets of paper, **copy the honor statement and sign.** Then write your answers on them, scan and submit onto Gradescope.

Good luck!

1. You do NOT need to provide explanation for the following questions.

(2) (a) True or False: $\frac{d}{dx}\log_3(5x) = \frac{d}{dx}\log_3 x$.

(3) (b) Choose ALL which are true about the series $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n}$.

A. The series absolutely converges
B. The series converges
C. Root test says the series converges
D. Alternating series test says the series converges
E. The series diverges

(3) (c) Suppose that the power series $\sum_{n=0}^{\infty} a_n(x-2)^n$ converges for $x = 5$. At which of the following points must it also converge? Choose ALL which are correct.

A. $x = -5$
B. $x = -4$
C. $x = -1$
D. $x = 1$
E. $x = 4$

(2) (d) True or False: If the Maclaurin series of an (infinitely differentiable) function $f(x)$ is 0, then $f(x) = 0$.

(2) (e) True of False: Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be two arbitrary series, where $\sum_{n=1}^{\infty} b_n$ is a <u>positive</u> series and <u>converges</u>. If $a_n \leq b_n$ for all $n \geq 1$, then $\sum_{n=1}^{\infty} a_n$ also converges.

(8) 2. Let $f(x) = 2x + \ln x$ for $x > 0$. Let $g(x)$ be the inverse function of $f(x)$. Compute $g'(2)$.

3. Evaluate the following improper integrals. If the integral diverges, show whether it diverges to ∞, $-\infty$ or neither.
(Hint: You will need to use integration by parts and partial fraction methods.)

(13) (a) $\displaystyle\int_0^1 \frac{\tan^{-1}(x)}{x^2}\,dx$

(12) (b) $\int_0^1 (\ln x)^2 \, dx$.

(10) 4. Determine whether the series

$$\sum_{n=3}^{\infty} \frac{(-1)^n}{(\ln n)^3}$$

absolute converges, conditionally converges, or diverges using any methods. You have to explicitly write down the methods you used to receive full credits.

(10) 5. Find the degree 2 Taylor polynomial $T_2(x)$ for $f(x) = \tan x$ centered at $x = \frac{\pi}{3}$.

(10) 6. Find the interval of convergence for $\sum_{n=2}^{\infty} \frac{x^n}{n^2 3^n}$.

(10) 7. Find the value of the series $\displaystyle\sum_{n=1}^{\infty} \frac{\pi - \cos n\pi}{4^n}$ or show that it diverges.

(10) 8. (a) Find the Maclaurin series of $\tan^{-1}(x)$. Your answer should be in terms of summation notation to receive full credits.

(5) (b) Find the value of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}\left(\frac{1}{\sqrt{3}}\right)^{2n+1}$ or show that it diverges.

(Hint: You can use the fact that $\tan^{-1} x$ is equal to its Maclaurin series on $(-1, 1)$ without justification.)