Midterm 2 Test Bank

Friday, February 18, 2022 4:01 AM

Midterm 2

Instructions:
Please write your answers and show all necessary work on either the provided exam (if you wish to print it), separate sheets of paper, or a tablet computer. You do not need to print out the exam. You may use your textbook, notes, and resources on CCLE or elsewhere on the internet. You may also use a calculator or computer, including online resources such as Desmos. However, as always, **you must show all of your work to receive full credit** for each problem, and **you must not get help from other people**, including by posting questions to an online forum.

All work shown must be your own. At the end of your exam, you must write and sign an honor statement to affirm this. (See question 5 below.)

If you have a question about the exam at any point during the exam period, the best option is to post your question to CampusWire, and select the option to "Post to instructors & TAs". You can also email your question to your instructor and/or TA. When you are finished with the exam, upload your answers to Gradescope. As usual, you have the option of uploading separate photos of each individual question, or uploading a single PDF file. We recommend that you start each question on a new page, as this makes it a little easier for the graders.

1. For each of the following functions, compute the specified derivative. The table to the right describes a function f, including its first and second derivatives. Use the values in the table to compute the derivatives specified below.

x	$f(x)$	$f'(x)$	$f''(x)$
-3	4	-5	1
-2	2	7	-2
-1	0	-5	-2
0	-3	-2	3
1	2	-6	8
2	-5	0	-3
3	9	6	-1
4	3	5	0

(a) (4 points) Compute $\left.\dfrac{d}{dx}\left(\dfrac{f(x^2)}{f(x)}\right)\right|_{x=-2}$

$$= \frac{f(x)f'(x^2)2x - f(x^2)f'(x)}{(f(x))^2} = \frac{f(-2)f'(4)2(-2) - f(4)f'(-2)}{(f(-2))^2} = \frac{2(5)(-4) - 3(7)}{2^2} = \frac{-40-21}{4} = \frac{-61}{4}$$

(b) (8 points) Let $g(x) = \sqrt{f(x)}$. Compute $g'(3)$ and $g''(3)$.

$$g(x) = (f(x))^{1/2}$$

$$g'(x) = \tfrac{1}{2}(f(x))^{-\frac{1}{2}} f'(x)$$

$$g'(3) = \tfrac{1}{2}(f(3))^{-\frac{1}{2}} f'(3) = \tfrac{1}{2}(9)^{-\frac{1}{2}}(6) = \tfrac{1}{2}\cdot\tfrac{1}{\sqrt{9}}\cdot 6 = \tfrac{6}{2\cdot 3} = \tfrac{6}{6} = \boxed{1} = g'(3)$$

$$g''(x) = -\tfrac{1}{4}(f(x))^{-\frac{3}{2}}\cdot f'(x)\cdot f'(x) + f''(x)\cdot\tfrac{1}{2}(f(x))^{-\frac{1}{2}}$$

$$g''(3) = \tfrac{-1}{4} f(3)^{-\frac{3}{2}}\cdot f'(3)\cdot f'(3) + f''(3)\cdot\tfrac{1}{2}(f(3))^{-\frac{1}{2}}$$

$$g''(3) = -\tfrac{1}{4}(9)^{-\frac{3}{2}}\cdot 6\cdot 6 + (-1\cdot\tfrac{1}{2}(9)^{-\frac{1}{2}})$$

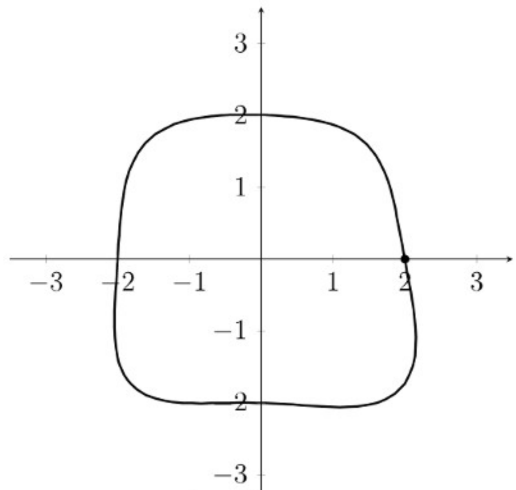
$$g''(3) = -\tfrac{1}{4}\cdot\tfrac{1}{\sqrt{9^3}}\cdot 36 + (-\tfrac{1}{2}\cdot\tfrac{1}{\sqrt{9}})$$

$$g''(3) = -\tfrac{1}{4}\cdot\tfrac{1}{27}\cdot 36 - \tfrac{1}{2\cdot 3} = -\tfrac{1}{108}\cdot 36 - \tfrac{1}{6} = \tfrac{-36}{108} - \tfrac{1}{6} = \boxed{-\tfrac{1}{2}} = g''(3)$$

2. (10 points) The equation

$$x^4 + x^2y + y^4 = 16 - x\sin(y)$$

defines a curve in the xy-plane, which passes through the point $(2, 0)$. Find an equation for the line tangent to this curve at the point $(2, 0)$.



(x,y)
$(2,0)$

$$x^4 + x^2y + y^4 = 16 - x\sin(y)$$

$4x^3 + x^2y' + y2x + 4y^3y' = -x\cos(y)y' + \sin(y)\cdot -1$

$4(8) + 4y' + 0 + 0 = -2\cos(0)y' - \sin(0)$

$32 + 4y' = -2y'$

$4y' + 2y' = -32$

$6y' = -32$

$y' = \frac{-32}{6} = \frac{-16}{3}$

$y' = \frac{-16}{3}$

$y - 0 = \frac{-16}{3}(x-2)$

3. (8 points) As you've probably heard, right now the world is in the midst of a global shortage of computer chips. The (fictional) company Initech has started producing their latest microprocessor, the Technium. From economics, we know that the price (P) of these processors will decrease as the supply (S) of them increases. As an industry analyst, you have created a model of this price versus supply relationship. Specifically, let S be the number of processors* the company produces each week. The table below gives the price P of one Technium processor, as well as the rate of change of that price $\frac{dP}{dS}$, for various values of S.

Since the company is currently ramping up their production of these processors, the supply S is increasing over time, according to the following function:

$$S(t) = \frac{30t^2}{6+t^2} \qquad \text{thousands of chips, } t \text{ in weeks}$$

How fast will the price be changing (and will it be increasing or decreasing) at time $t = 2$? find $\frac{dP}{dt}$ at $t=2$

Note: For full credit be sure to indicate which value(s) from the table you used, and specify units for your answer.

Chips produced S (thousands)	Price P (\$)	Rate of change of price $\frac{dP}{dS}$ (\$ per thousand chips)
1	250	−17.50
2	225	−10.95
5	190	−5.50
8	175	−3.60
10	160	−2.80
12	155	−4.50 I used this value
15	140	−4.20
20	120	−2.30
30	100	−1.80

*S is measured in thousands of processors, but you can completely ignore that for the purposes of this problem.

$$\frac{dP}{dt} = \frac{dS}{dt} \cdot \frac{dP}{dS}\Big|_{S=S(t)} \quad S(2) = \frac{30(4)}{6+4} = \frac{120}{10} = 12$$

$$\frac{dS}{dt} = S' = \frac{d}{dt}\left(\frac{30t^2}{6+t^2}\right) = \frac{(6+t^2)(60t)-(30t^2)(2t)}{(6+t^2)^2}$$

$$S'(2) = \frac{(6+4)(120)-(30\cdot 4)(4)}{(6+4)^2} = \frac{10(120)-(120)(4)}{10^2} = \frac{1200-480}{100} = \frac{720}{100} = \frac{36}{5}$$

$$\frac{d}{dt} P(S(t)) = P'(S(t)) \cdot S'(t) = P'(S(2)) \cdot S'(2) = P'(12) \cdot \frac{36}{5} = \frac{dP}{dS}\Big|_{S=12} \cdot \frac{36}{5} = -4.5 \cdot \frac{36}{5} = -32.4$$

The price will be changing at a rate of \$32.4/week. It will be decreasing.

4. (10 points) Two well known formulas from physics† state that for an object of mass m, if it is moving at a speed v then its kinetic energy is $E_{\text{kin}} = \frac{1}{2}mv^2$, and if its height is h then its potential energy due to gravity is $E_{\text{pot}} = mgh$, where $g = 9.8\,\frac{\text{m}}{\text{s}^2}$. Therefore the total mechanical energy of the object is

$$E = \frac{1}{2}mv^2 + 9.8mh$$

$(6+4)^2$ 10^2 100 100 5

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$$E = \frac{1}{2}mv^2 + 9.8mh$$

A model rocket builder has launched a rocket, and has equipped it with sensors to measure its altitude and speed as it travels up through the air. Since the rocket's fuel makes up a significant portion of its mass, and it burns through all of that fuel in a matter of seconds, the mass of this object (m) is changing significantly. And of course, its speed (v) and height (h) are also changing over time.

At a certain instant in its flight, the sensors on the model rocket tell you the following:

- At that instant, the total mass of the rocket is 12 kg. $m=12$
- The rocket is using up its fuel at a rate of $1.3\,\frac{\text{kg}}{\text{s}}$, so the total mass of the rocket is *decreasing* at this same rate. $\frac{dm}{dt} = -1.3$
- The rocket is 200 m above the ground. $h=200$
- The rocket is moving upward at a speed of $45\,\frac{\text{m}}{\text{s}}$. $\frac{dh}{dt} = 45$ $v=45$
  (Hint: Since the rocket is going straight up, this speed is equal to both v, and the rate of change of h.)
- The rocket is accelerating upward at a rate of $7\,\frac{\text{m}}{\text{s}^2}$. $\frac{dv}{dt} = 7$

Find the rate of change of the total energy of the rocket (E from the big equation above) at the instant just described. Include the correct unit(s) in your final answer for full credit. find $\frac{dE}{dt}$

(Note: The rate of change of energy is *power*, so the quantity you've just calculated is the total power being imparted to the rocket by its engine. This is not important for solving the problem, but it might help to put the problem into familiar terms.)

work and answer on next page (after honor statement)

†You don't need to know anything about these individual equations for this problem. You can just use the big equation given above, exactly as it's written.

5. (0 points) Please write the following honor statement and sign your name after it.

I certify on my honor that I have neither given nor received any help, nor used any non-permitted resources, while completing this evaluation.

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4. variables: E, m, v, h

givens: $m = 12$

$\frac{dm}{dt} = -1.3$

$h = 200$

$\frac{dh}{dt} = 45$

$v = 45$

$\frac{dv}{dt} = 7$

find: $\frac{dE}{dt}$

Solve: $E = \frac{1}{2}mv^2 + 9.8mh$

$E' = \frac{1}{2}m2vv' + v^2\frac{1}{2}m' + 9.8mh' + h9.8m'$

$E' = \frac{1}{2}(12)(2)(45)(7) + (45^2)(\frac{1}{2})(-1.3) + 9.8(12)(45) + (200)(9.8)(-1.3)$

$E' = 5{,}207.75 \ J/s$ $= 5{,}207.75$ watts