19F-MATH31AL-2 Midterm 1

CHRISTIAN AGUILAR

TOTAL POINTS

45 / 50

QUESTION 1

Compute some limits 15 pts

1.1 $\infty - \infty$: Combine fractions 5 / 5

- **1 pts** Computational error
- **2 pts** Two or more computational errors
- **2 pts** One invalid algebraic manipulation
- **4 pts** Multiplie invalid algebraic manipulations
- **3 pts** The student believes that a 0 denominator automatically implies the limit is infinite
- **2 pts** Correct steps but the cancellation of s and evaluation of the limit are missing
- **5 pts** No progress toward a solution

✓ **- 0 pts Correct with correct steps shown**

1.2 0/0: Multiply top and bottom by conjugate 5 / 5

✓ **- 0 pts Correct answer with steps shown clearly**

- **1 pts** Computational error
- **2 pts** Multiple computational errors
- **1 pts** The student correctly manipulates the expression to a point where substitution is valid, but does not evaluate the limit
- **2 pts** The student correctly rationalizes and simplifies the denominator but does not proceed further
- **3 pts** The student multiplies by the conjugate/conjugate but does not proceed further or simplifies the denominator incorrectly and then stops
- **5 pts** No progress toward a solution

1.3 1/0: Infinite limits; check limits from left and right 5 / 5

✓ **+ 1 pts The student unambiguously identifies that the limit does not exist (either stating this or saying it is infinite, but not claiming that is it equal to +/- infinity or 1/0). Or the student mistakenly finds that the left and right-hand limits are equal and uses this to say that the limit is their common value.**

✓ **+ 2 pts Left-hand limit of -infinity with justification.**

✓ **+ 2 pts Right-hand limit of +infinity with justification.**

+ **1 pts** Left-hand limit of -infinity with incorrect or no justification.
+ **1 pts** Right-hand limit of +infinity with incorrect or no justification.
+ **3 pts** The student mixes up the signs of sin(x) to the left versus right of 0, but their sided limits are consistent with this mistake.
+ **0 pts** None of the limits are clearly and correctly stated.

Definitely, not probably

QUESTION 2

2 Continuity for a piecewise function 12 / 12

✓ **- 0 pts Correct, a = 7 and "no" + justification.**

- **1 pts** Part a) Mistake when solving for "a"
- **2 pts** Part a) Mistake when factoring top
- **2 pts** Part a) Mistake when factoring bottom
- **1 pts** Part a) Mistake evaluating pieces at x = 1
- **7 pts** Part a) No significant progress toward solution
- **1 pts** Part b) Incorrect conclusion
- **2 pts** Part b) Justification incorrect, unclear, or missing
- **1 pts** Part b) Missing a significant amount of work
- **5 pts** Part b) No significant progress made toward solution
- **4 pts** Part b) Attempt to equate wrong pieces of the function at x = 3, leading to incorrect conclusion

- **2 pts** Part a) Using l'hopital's rule doesn't show that the material covered in class is understood

QUESTION 3

3 Rates of change, and limit definition of the derivative 8 / 13

✓ **+ 2 pts** **Correct difference quotient**

✓ **+ 1 pts** **Correct answer in (a)**

+ **5 pts** Correct definition of the derivative in (b)
+ **5 pts** Correct derivative computation in (b)
+ **1 pts** Numerator only in (a)

✓ **+ 4 pts** **Almost correct derivative definition in (b)**

+ **4 pts** Minor error in computation for (b)

✓ **+ 1 pts** **Marginal partial credit for (b)**

+ **2 pts** Partial credit for (b)
+ **3 pts** Partial credit for (b)
+ **1 pts** Wrong sign in difference quotient for (a)
+ **0 pts** Incorrect

QUESTION 4

Computing derivatives using differentiation rules 10 pts

4.1 Compute slope of tangent line (power rule) 5 / 5

✓ **- 0 pts** **Correct**

- **1 pts** Correct derivative, minor error plugging in 4.
- **3 pts** Differentiated sqrt(x) correctly, other derivatives wrong.
- **2 pts** Differentiated only the first two terms correctly.
- **5 pts** Nothing substantive

4.2 Compute instantaneous rate of change (product rule) 5 / 5

✓ **- 0 pts** **Correct**

- **1 pts** Correct derivative, minor error plugging in 1
- **2 pts** Power rule error with t^{-1}
- **3 pts** Took correct derivatives of both factors, did not apply product rule.
- **4 pts** Took just one correct derivative
- **5 pts** Nothing substantive

Midterm 1
Version A

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Section:

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Instructions: Do not open this exam until instructed to do so. You will have 50 minutes to complete the exam. Please print your name and student ID number above, and circle the number of your discussion section. **You may not use calculators,** books, notes, or any other material to help you. Please make sure **your phone is silenced** and stowed where you cannot see it. You may use any available space on the exam for scratch work. If you need more scratch paper, please ask one of the proctors. You must **show your work** to receive credit. Please circle or box your final answers.

Please do not write below this line.

Question	Points	Score
1	15	
2	12	
3	13	
4	10	
Total:	50	

1. Compute the following limits. If a limit does not exist, be as specific as possible. (E.g. for an infinite limit, find the one-sided limits.) As always, you must justify each answer.

(a) (5 points) $\lim_{s\to 0}\left(\frac{s+12}{s^2+3s}-\frac{4}{s}\right)$ $= \frac{s+12}{s(s+3)} - \frac{4(s+3)}{s(s+3)}$

Try to plug in: $\frac{0+12}{0+0} - \frac{4}{0} = \frac{s+12-4(s+3)}{s(s+3)}$ evaluates to $\frac{0}{0}$, indeterminate!

$\to \frac{s+\cancel{12}-4s-\cancel{12}}{s(s+3)}$ $\frac{s-4s}{s(s+3)} \to \frac{-3s}{s(s+3)}$

$\to \lim_{s\to 0} \frac{-3}{(s+3)} \to \frac{-3}{0+3} \to \boxed{-1}$

check $\frac{1}{-(16-t)^{-\frac{1}{2}}-(t+2)^{-\frac{1}{2}}}$

$-\frac{1}{2}(16 - \sqrt{16-t} - \frac{1}{2}\sqrt{16-t}$

$-\frac{1}{2}\sqrt{9} - \frac{1}{2}\sqrt{9}$

(b) (5 points) $\lim_{t\to 7}\frac{t-7}{\sqrt{16-t}-\sqrt{t+2}}$

Try plugging in: $\lim_{t\to 7}\frac{7-7}{\sqrt{16-7}-\sqrt{7+2}} \to \frac{0}{\sqrt{9}-\sqrt{9}} \to \frac{0}{0} \to$ indeterminate,

$\frac{t-7}{\sqrt{16-t}-\sqrt{t+2}} \cdot \frac{\sqrt{16-t}+\sqrt{t+2}}{(\sqrt{16-t}+\sqrt{t+2})}$

$\to \frac{(t-7)(\sqrt{16-t}+\sqrt{t+2})}{16-t-t-2} \to \frac{(t-7)(\sqrt{16-t}+\sqrt{t+2})}{-2t+14}$

$\to \frac{\cancel{(t-7)}(\sqrt{16-t}+\sqrt{t+2})}{-2\cancel{(t-7)}} \to -\frac{(\sqrt{16-t}+\sqrt{t+2})}{2}$

$-\frac{(\sqrt{16-7}+\sqrt{7+2})}{2}$

$-\left(\frac{3+3}{2}\right) \to \boxed{-3}$

Question 1 continues on the next page...

Question 1 continued...

(c) (5 points) $\lim_{x \to 0} \dfrac{x+1}{\sin(x)}$

Try plugging in: $\dfrac{0+1}{\sin(0)} \rightarrow \dfrac{1}{0}$: probably infinite limit!

$\lim_{x \to 0^+}$ approaches $+\infty$ because the denominator will evaluate to some very small positive number.

$\lim_{x \to 0^-}$ approaches $-\infty$ because the denominator $(\sin(x))$ approaches some tiny negative value.

2. (12 points) Define a function f as follows, where a and b are unknown constants:

$$f(x) = \begin{cases} \frac{ax+3}{x-5} & \text{if } x \leq 1 \\ \frac{x^3+x^2-2}{x^2-4x+3} & \text{if } 1 < x < 3 \\ x^2 - b & \text{if } x \geq 3 \end{cases}$$

→ we probably don't care about this...

(a) Find the value of the constant a so that f will be continuous at $x = 1$, if this is possible. If not, explain why not.

Continuity: $f(1)$ must be defined and exist

- $\lim_{x\to 1} f(x)$ must exist
- $\lim_{x\to 1} f(x) = f(1)$

$\frac{ax+3}{x-5} =$

$\lim_{x\to 1} \frac{x^3+x^2-2}{x^2-4x+3}$

$\frac{1+1-2}{1-4+3} \to \frac{0}{0}$, ind. form, removable discontinuity

$x^2-4x+3 = x^2-1x-3x+3$

$x^2-3x \quad -1x+3$

$x(x-3)-1(x-3)$

$\to \frac{x^3+x^2-2}{(x-1)(x-3)}$

$x-1 \,\overline{)\, x^3+x^2+0x-2}$, quotient x^2+2x+2

x^3-x^2

$2x^2+0x$

$2x^2-2x$

$2x-2$

$2x-2$

0

$\lim_{x\to 1} \frac{(x^2+2x+2)\cancel{(x-1)}}{(x-3)\cancel{(x-1)}}$

$\lim_{x\to 1} \frac{(x^2+2x+2)}{(x-3)}$

$\to \frac{1^2+2(1)+2}{1-3} \to \boxed{-\frac{5}{2}}$

$\frac{a(1)+3}{1-5} = -\frac{5}{2}$

$\frac{a+3}{-4} = -\frac{5}{2}$

$a+3 = \frac{20}{2}$

$a+3 = 10$

$a = 7$

$\frac{10}{-4} = -\frac{5}{2}$

Question 2 continues on the next page...

Question 2 continued...

(b) Find the value of the constant b so that f will be continuous at $x = 3$, if this is possible. If not, explain why not.

$$\frac{x^3-x^2-2}{x^2-4x+3} \to \frac{3^3-3^2-2}{3^2-4(3)+3} \quad \frac{27-9-2}{3^2-4(3)+3}$$

$$\to \frac{18-2}{9-12+3} \to \frac{16}{0}$$

This is not possible because The limit as $x \to 3$ of $\frac{x^3-x^2-2}{x^2-4x+3}$ approaches some infinity for any value of b. Since there is no actual existing, defined limit at $f(x) \to 3$, by definition it can not be continuous.

3. When a car applies the brakes to come to a stop, its position at time t is given by

$$p(t) = \frac{t^2}{1+t^2}$$

Use this function to answer the following:

(a) (3 points) Find the average rate of change of the car's position (average speed) over the interval from $t = 1$ to $t = 3$.

avg rate of change $= \dfrac{p(3) - p(1)}{3-1}$

$p(3) = \dfrac{3^2}{1+3^2} \rightarrow \dfrac{9}{10}$

$p(1) = \dfrac{1}{1+1} \rightarrow \dfrac{1}{2} \rightarrow \dfrac{\frac{9}{10} - \frac{1}{2}}{2} \rightarrow \dfrac{\frac{9}{10} - \frac{5}{10}}{2}$

$\rightarrow \dfrac{4}{10} \cdot \dfrac{1}{2} \rightarrow \boxed{\dfrac{2}{10}}$ or $\dfrac{1}{5}$ $\quad \dfrac{4}{20}$

(b) (10 points) Find the instantaneous rate of change of the car's position (the actual speed) at $t = 1$. *Use the limit definition for this, not just differentiation rules.*

$p'(1) = \lim_{h \to 0} \dfrac{\cancel{p(x) - p(x+h)}}{h} \quad \dfrac{p(x+h) - p(x)}{h}$

$\lim_{h \to 0} \rightarrow \dfrac{\dfrac{(t+h)^2}{1+(t+h)^2} - \dfrac{t^2}{(1+t^2)}}{h} \qquad + \quad 1+t^2$

$\dfrac{\dfrac{(1+t^2)(t+h)^2 - (t^2(1+t+h^2))}{(1+t^2)\ 1+(t+h)^2}}{h}$

Question 3 continues on the next page...

Question 3 continued...

$$\frac{\frac{(x+h)^2}{1+(x+h)^2} - \frac{x^2}{(1+x^2)}}{h}$$

$$\frac{x^2 + 2xh + x^2}{1+ x^2 + 2xh^2 + x^2} - \frac{x^2}{1+x^2}$$

$$\frac{\frac{(1+x^2)(x+h)^2}{(1+x^2)(1+(x+h)^2)} - \frac{x^2(1+(x+h)^2)}{(1+x^2)(1+(x+h)^2)}}{h}$$

$$\frac{(1+x^2)(x+h)^2 - x^2(1+(x+h)^2)}{h(1+x^2)(1+(x+h)^2)}$$

$\cancel{x^2} + x^4 + 2xh + 2x^3h + h^2 + x^2h^2$

$-$

$1 \quad x^2$

$x^2 \quad x^2 \quad x^4$

$2xh \quad 2x^3h$

$2xh \quad 2x^3h$

$h^2 \quad h^2 \quad x^2h^2$

$x^2 \quad 2xh \quad h^2 \quad 1$

$x^4 \quad 2x^3h \quad x^2h^2 \quad x^2$

$x^2 \quad x^4$

4. (a) (5 points) Let $f(x) = 12\sqrt{x} - \frac{2}{x^2} + \frac{1}{\sqrt{x}} - 11$.

Find the slope of the tangent line to the graph of $f(x)$ at $x = 4$.

$f(t) = 12\sqrt{x} - \frac{2}{x^2} + \frac{1}{\sqrt{x}} - 11$

$= 12x^{\frac{1}{2}} - 2x^{-2} + x^{-\frac{1}{2}} - 11$

$f'(t) = (\frac{1}{2})12x^{-\frac{1}{2}} - 2(-2)x^{-3} + (-\frac{1}{2})x^{-\frac{3}{2}}$

$f'(t) = 6x^{-\frac{1}{2}} + 4x^{-3} - \frac{1}{2}x^{-\frac{3}{2}}$

$f'(4) = \frac{6}{\sqrt{4}} + \frac{4}{4^3} - \frac{1}{2 \cdot 4^{\frac{3}{2}}}$

$= \frac{6}{2} + \frac{4}{64} - \frac{1}{2 \cdot 8}$

$= 3 + \frac{1}{16} - \frac{1}{16}$

$\boxed{= 3}$

(b) (5 points) Let $g(t) = (t + t^{-1}) \cdot (3t^2 - 5t + 7)$.

Find the instantaneous rate of change of g at $t = 1$.

$g'(t) = (t+t^{-1})' \cdot (3t^2-5t+7) + (t+t^{-1})(3t^2-5t+7)'$

$(1-t^{-2})(6t-5) + (t+t^{-1})$

$(1-t^{-2})(3t^2-5t+7) + (t+t^{-1})(6t-5)$

$g'(t)$

$g'(1) = (1-1^{-2})(3(1)^2-5(1)+7) + (1+1^{-1})(6(1)-5)$

$\rightarrow$ (1-1) 0

$2(6-5)$

$2(1)$

$\boxed{= 2}$