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Midterm 2

1)

$f(x) = \tan\left(\frac{x}{x^2+1}\right)$

$f'(x) = \sec^2\left(\frac{x}{x^2+1}\right) \cdot \left(\frac{(x^2+1)(1) - (x)(2x)}{(x^2+1)^2}\right)$

$= \sec^2\left(\frac{x}{x^2+1}\right) \cdot \frac{x^2+1-2x^2}{(x^2+1)^2}$

$= \boxed{\frac{\sec^2\left(\frac{x}{x^2+1}\right)(-x^2+1)}{(x^2+1)^2}}$

2) $g(x) = \sin x$

$g'(x) = \cos x$

$g''(x) = -\sin x$

$g'''(x) = -\cos x$

$g^{(4)}(x) = \sin x$

$g^{(5)}(x) = \cos x$

$g^{(6)}(x) = -\sin x$

$g^{(7)}(x) = -\cos x$

$g^{(8)}(x) = \sin x$

repeats every 4 iterations

$g^{(4n)}(x) = \sin x$ ← if Remainder = 0

$g^{(4n+1)}(x) = \cos x$ ← if R = 1

$g^{(4n+2)}(x) = -\sin x$ ← if R = 2

$g^{(4n+3)}(x) = -\cos x$ ← if R = 3

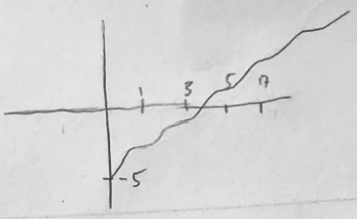
$g^{(100)}(x) =$

100/4 = 25, remainder = 0, so $g^{(100)}(x) = \sin x$

$g^{(103)}(x)$

10314 = 25, remainder = 3, $\boxed{\text{so } g^{(103)}(x) = -\cos x}$

f prob looks like



3) a) f does not achieve any local maxes b/c the sign of f' does not change before and after crit points

b) f does not achieve any local mins for the same reason ↗

c) yes it is possible (endpoints)

d) None (open interval)

e) $x=5$

4) $V = \pi r^2 \cdot h$ $\quad r = 10\,nm$ $\quad h = 20\,nm$

$\frac{dV}{dt} = 300\ nm^3/s$ $\quad \frac{dr}{dt} = 2 \cdot \frac{dh}{dt}$ $\quad \frac{dh}{dt} = ?$

$$\frac{dV}{dt} = \left(2\pi r \frac{dr}{dt}\right)(h) + (\pi r^2)\frac{dh}{dt}$$

$$300 = \left(2\pi 10 \cdot 2\frac{dh}{dt}\right)(20) + (\pi (10)^2)\frac{dh}{dt}$$

$$300 = 40\pi \frac{dh}{dt}(20) + 100\pi \frac{dh}{dt}$$

$$300 = \frac{dh}{dt}(800\pi + 100\pi)$$

$$\frac{300}{900\pi} = \frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt} = \frac{1}{3\pi}\ nm/s} \qquad \frac{dr}{dt} = 2 \cdot \frac{1}{3\pi} = \boxed{\frac{2}{3\pi}\ nm/s}$$

5) $S(c) = \sqrt[3]{c+3} + 2c - 4$ $\quad c=5,\ S=8$

$\Delta S = S'(c)(\Delta c)$

$2 = \left(\frac{1}{3}(c+3)^{-\frac{2}{3}} + 2\right)(\Delta c)$

$2 = \left(\frac{1}{3}(8)^{-\frac{2}{3}} + 2\right)(\Delta c)$

$2 = \left(\frac{1}{3 \cdot 4} + 2\right)\Delta c$

$\frac{2}{\left(\frac{1}{12}+2\right)} = \Delta c$

$\frac{2}{\frac{25}{12}} = \Delta c$

$\boxed{\Delta c = \frac{24}{25}}$

6) • $f(x)$ cont, diff $\quad f(4) \neq 5$

• $f'(x) \geq 3$ everywhere

• $f(2) = 1$

MVT says there should be a point in this function where $f'(c) = \frac{f(b)-f(a)}{a-b}$

$\frac{f(4)-f(2)}{4-2} = \frac{5-1}{4-2} = \frac{4}{2} = 2$

↑

but $f'(x)$ is supposed to be ≥ 3 everywhere, therefore $f(4)$ cant be equal to 5 since 2 is not ≥ 3