MATH 31A

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1a) $\lim_{x\to4} \frac{x^2-16}{x^2-5x+4} \Rightarrow \lim_{x\to4} \frac{(x-\cancel{4})(x+4)}{(x\cancel{-4})(x-1)}$

$= \frac{4+4}{4-1} = \boxed{\frac{8}{3}}$

1b) $\lim_{x\to0} \frac{(x+6)\sin(5x)}{\tan(3x)} = \lim_{x\to0}(x+6) \cdot \lim_{x\to0} \frac{\sin(5x)}{\frac{\sin(3x)}{\cos(3x)}}$

$= 6 \cdot \lim_{x\to0} 5x \cdot \frac{\sin(5x)}{5x} \cdot \frac{\cos(3x)}{\sin(3x)}$

$= 6 \cdot \lim_{x\to0} 5x \cdot \frac{\sin(5x)}{(5x)} \cdot \frac{\cos(3x)}{3x \cdot \frac{\sin(3x)}{3x}}$

$= 6 \cdot \lim_{x\to0} \frac{5\cancel{x}}{3\cancel{x}} \cdot \frac{\sin(5x)}{(5x)} \cdot \frac{\cos(3x)}{\sin(3x)/3x}$

$= 6 \cdot \frac{5}{3} \cdot 1 \cdot \frac{1}{1}$

$= \frac{30}{3} = \boxed{10}$

1c) $\lim_{x\to\infty} \sqrt{x^4+x^2} - x^2$

$= \lim_{x\to\infty} \frac{x^4+x^2-x^4}{\sqrt{x^4+x^2}+x^2}$

$= \lim_{x\to\infty} \frac{x^2}{x^2\left(\sqrt{1+\frac{1}{x^2}}+1\right)} \Rightarrow \lim_{x\to\infty} \frac{1}{\sqrt{1+\frac{1}{x^2}}+1}$

$= \frac{1}{\sqrt{1+0}+1} = \boxed{\frac{1}{2}}$

1d) $\lim_{x \to 8} (x-8)^2 \cos\left(\cos\left(\frac{1}{x-8}\right)\right)$

$= -1 \le \cos\left(\cos\frac{1}{x-8}\right) \le 1$

$-(x-8)^2 \le (x-8)^2 \cos\left(\cos\frac{1}{x-8}\right) \le (x-8)^2$

$\lim_{x \to 8} -(x-8)^2 \le \lim_{x \to 8} (x-8)^2 \cos\left(\cos\frac{1}{x-8}\right) \le \lim_{x \to 8} (x-8)^2$

$0 \le \lim_{x \to 8} (x-8)^2 \cos\left(\cos\frac{1}{x-8}\right) \le 0$

$\boxed{\text{Therefore by squeeze theorm, } \lim_{x \to 8} (x-8)^2 \cos\left(\cos\left(\frac{1}{x-8}\right)\right) = 0}$

2a) False
b) False
c) True
d) False
e) True

3) $g(x) = \frac{1}{x+1}$

$\lim_{h \to 0} \frac{\left(\frac{1}{3+h+1}\right) - \frac{1}{3+1}}{h} = \lim_{h \to 0} \frac{\frac{1}{4+h} - \frac{1}{4}}{h}$

$= \lim_{h \to 0} \frac{\frac{4}{4(4+h)} - \frac{4+h}{4(4+h)}}{h}$

$= \lim_{h \to 0} \frac{\frac{4-4-h}{4(4+h)}}{h}$

$= \lim_{h \to 0} \frac{-\cancel{h}}{16+4h} \cdot \frac{1}{\cancel{h}} = \lim_{h \to 0} \frac{-1}{16+4(0)} = \boxed{\frac{-1}{16}}$

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4a) $f(-3) = 3$

b) $f'(-3) = -\frac{1}{2}$

c) $f(4) = 0$

d) $f'(4) =$ DNE

5) $g(2)=3 \quad g'(2) = -1$

$$\frac{dy}{dx} = 3 \cdot g'(x) - (4 \cdot -1x^{-2})$$

$$\frac{dy}{dx} = 3 \cdot (-1) - (4 \cdot -\frac{1}{4})$$

$$= -3 + 1 = -2$$

$$y = 3(g(2)) - \frac{4}{2}$$

$$y = 3(3) - 2$$

$$y = 7$$

$$\boxed{y - 7 = -2(x-2)}$$