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FINAL MATH 31A

1) a) $\lim_{x\to 2} \frac{\sqrt{x^2+5}-3}{x-2} \cdot \frac{(\sqrt{x^2+5}+3)}{(\sqrt{x^2+5}+3)}$

$\lim_{x\to 2} \frac{x^2+5-9}{(x-2)(\sqrt{x^2+5}+3)}$

$\lim_{x\to 2} \frac{x^2-4}{(x-2)(\sqrt{x^2+5}+3)}$

$\lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(\sqrt{x^2+5}+3)} \Rightarrow \frac{2+2}{\sqrt{2^2+5}+3} = \frac{4}{6} = \boxed{\frac{2}{3}}$

b) $\lim_{x\to\infty} \frac{3x^2-\sqrt{x^4+x}}{8x^2+5}$

$\lim_{x\to\infty} \frac{3-\sqrt{1+x^{-3}}}{8+5x^{-2}}$

$= \frac{3-\sqrt{1+0}}{8+0} = \frac{2}{8} = \boxed{\frac{1}{4}}$

2) a) $f(x) = \tan(x^3+\sqrt{x})$

$\boxed{f'(x) = \sec^2(x^3+\sqrt{x}) \cdot (3x^2 + \tfrac{1}{2}x^{-\frac{1}{2}})}$

b) $g(x) = \frac{x}{\cos x + \csc x}$

$\boxed{g'(x) = \frac{(\cos x + \csc x)(1) - (x)(-\sin x - \csc x \cot x)}{(\cos x + \csc x)^2}}$

3) $xy + y^2 = \int_6^x t^t\, dt$ (d/dx both sides)

$y + \frac{dy}{dx}x + 2y \cdot \frac{dy}{dx} = x^x \cdot 1$

$\frac{dy}{dx}x + \frac{dy}{dx}2y = x^x - y$

$\frac{dy}{dx}(x+2y) = x^x - y$

$\boxed{\frac{dy}{dx} = \frac{x^x - y}{(x+2y)}}$

4) a) $\int \left(4x - 20x^3 + \frac{x^3}{\sqrt{x}}\right) dx$

$= 4\int (x)\,dx - 20\int (x^3)\,dx + \int \left(\frac{x^3}{\sqrt{x}}\right) dx$

$= \frac{4}{2}x^2 - \frac{20}{4}x^4 + \int (x^{5/2})\,dx + C$

$= \boxed{2x^2 - 5x^4 + \frac{2}{7}x^{7/2} + C}$

b) $\int_{\pi/2}^{\pi} \sin\left(\frac{5\theta - \pi}{6}\right) d\theta$

$u = \frac{5\theta - \pi}{6}$

$du = \frac{5}{6}\,d\theta$

$\frac{6}{5}\int_{\theta=\pi/2}^{\theta=\pi} \cancel{\sin(u)\,du}$

$= \frac{6}{5}\left(-\cos(u)\right)\Big|_{\theta=\pi/2}^{\theta=\pi}$

$= \frac{6}{5}\left(-\cos\left(\frac{5\pi-\pi}{6}\right) + \cos\left(\frac{5\pi/2-\pi}{6}\right)\right)$

$\frac{6}{5}\left(-\left(-\frac{1}{2}\right) + \frac{1}{\sqrt{2}}\right)$

$\approx \frac{6}{10} + \frac{6}{5\sqrt{2}} \approx \boxed{1.4185}$

5) a) $x^2 - 9$ when $x = 3$ is 0

$\int_a^{x^2} \sqrt{\sqrt{t} + b}\,dt = 0$ would make $f(x)$ continuous,

which happens when $a = x^2$, so $\boxed{a = 9}$

b) $\frac{d}{dx}\, x^2 - 9 = 2x$, at $x = 3$ derivative is 6

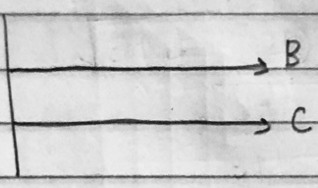
$\frac{d}{dx}\int_9^{x^2} \sqrt{\sqrt{t} + b}\,dt = 6$ makes $f'(x)$ continuous everywhere

$\sqrt{\sqrt{x^2} + b} \cdot 2x = 6$

$\sqrt{3 + b} \cdot 2(3) = 6$

so $\boxed{b = -2}$

6)



$B'(0) = 15\ m/s \quad B''(t) = 4\ m/s^2$

$C'(0) = 0\ m/s \quad C''(t) = 6t\ m/s^2$

$B''(t) = 4\ m/s^2 \quad \int 4\,dt$

$B'(t) = 4t + C$

$15 = 4(0) + C$

$C = 15$

$\boxed{B'(t) = 4t + 15} \quad \int 4t + 15\,dt$

$B(t) = \frac{4}{2}t^2 + 15t + C$

$0 = 2(0)^2 + 15(0) + C$

$C = 0$

$\boxed{B(t) = 2t^2 + 15t}$

$C''(t) = 6t\ m/s^2 \quad \cancel{\int 6t\,dt}$

$C'(t) = \frac{6}{2}t^2 + C$

$0 = 3(0)^2 + C$

$C = 0$

$\boxed{C'(t) = 3t^2} \quad \int 3t^2\,dt$

$C(t) = \frac{3}{3}t^3 + C$

$0 = t^3 + C$

$C = 0$

$\boxed{C(t) = t^3}$

$2t^2 + 15t = t^3$

$2t + 15 = t^2$

$0 = t^2 - 2t - 15$

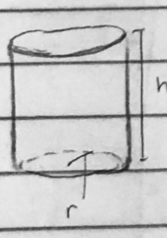
$0 = (t-5)(t+3)$

$t = 5, -3$

out of range

$\boxed{\text{Carla catches up to Bob at } t = 5 \text{ sec.}}$

7)



$V = \pi r^2 h = 27\pi\ cm^3$

$SA = \pi r^2 + 2\pi r(h)$

$h = \frac{27\pi}{\pi r^2}$

$\boxed{SA = \pi r^2 + 2\pi r\left(\frac{27\pi}{\pi r^2}\right)}$

$SA = \pi r^2 + \frac{54\pi}{r}$

$SA' = 2\pi r + 54\pi(-1r^{-2})$

Interval:

$r > 0$ because $h = \frac{27\pi}{\pi r^2}$

$h \geq 0$

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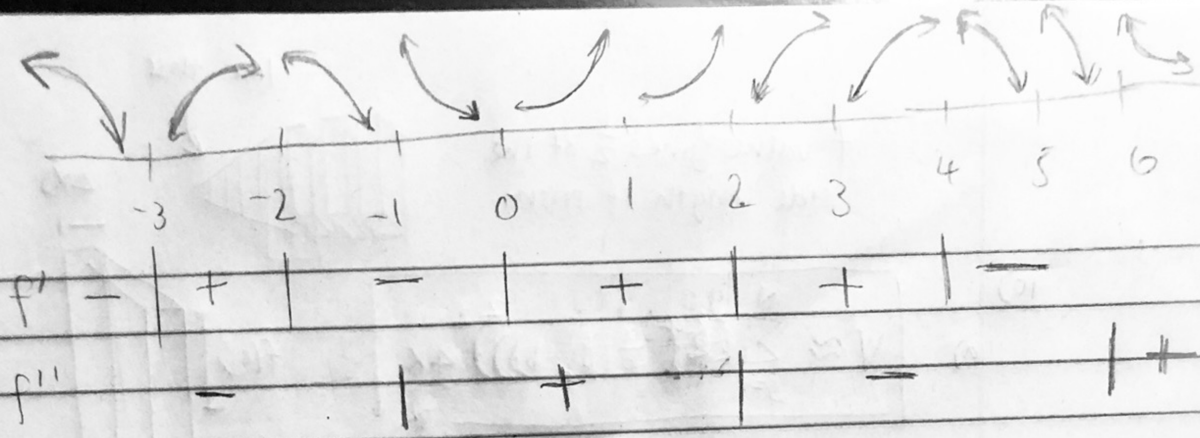
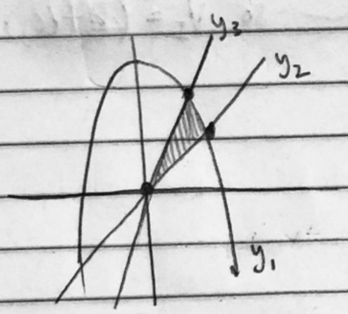


b) the critical point @ $x=-3$, f' is undefined

9) a)



$y_1 = 6 - x^2$

$y_2 = x$

$y_3 = 5x$

$6 - x^2 = 5x$

$0 = x^2 + 5x - 6$

$0 = (x+6)(x-1)$

$x = -6 \quad \boxed{x=1}$

$6 - x^2 = x$

$0 = x^2 + x - 6$

$0 = (x+3)(x-2)$

$x = -3, \quad \boxed{x=2}$

$$\boxed{\int_0^1 (5x - x)\,dx + \int_1^2 \left((6-x^2) - x\right) dx}$$

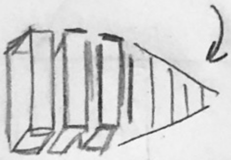
b)

$$\boxed{\pi \int_0^1 \left((5x)^2 - x^2\right) dx + \pi \int_1^2 \left((6-x^2)^2 - x^2\right) dx}$$

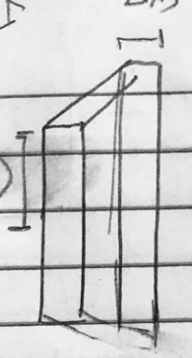
c)

$$\boxed{2\pi \int_0^1 x(5x - x)\,dx + 2\pi \int_1^2 x\left((6-x^2) - x\right) dx}$$

like this



f value gives $\frac{1}{2}$ of the side length of prism



10)

a) $$\boxed{V \approx \sum_{j=1}^{N} \left(2\left(-\frac{a}{b}(c_j - b)\right)\right)^2 \Delta x}$$

so $V_I = (2f(c_j))^2 \Delta x_j$

b)

$$\int_0^b \left(\left(2\left(-\frac{a}{b}(x-b)\right)\right)^2\right) dx$$

$$4\int_0^b \left(-\frac{a}{b}(x-b)\right)^2 dx$$

$$4\int_0^b \left(-\frac{ax}{b} - \frac{ab}{b}\right)^2 dx$$

$$4\int_0^b \left(\frac{a^2}{b^2}x^2 - \frac{2a^2x}{b} + a^2\right) dx$$

$$4\left(\int_0^b \frac{a^2}{b^2}x^2 dx - \int_0^b \frac{2a^2x}{b} dx + \int_0^b a^2 dx\right)$$

$$4\left(\frac{a^2b}{3} - a^2b + a^2b\right)$$

$$4\left(\frac{a^2b}{3}\right)$$

↓

So Vol can be written as

$$\boxed{V = \frac{4}{3}a^2b}$$

$$0 = 2\pi r + 54\pi\left(-\frac{1}{r^2}\right)$$

$$\frac{54\pi}{r^2} = 2\pi r$$

$$54\pi = 2\pi r^3$$

$$27 = r^3$$

CRIT POINTS: $\boxed{r=3}$ SA' DNE at r=0, but r has to be > 0 so r=0 is not in range

MIN or MAX:

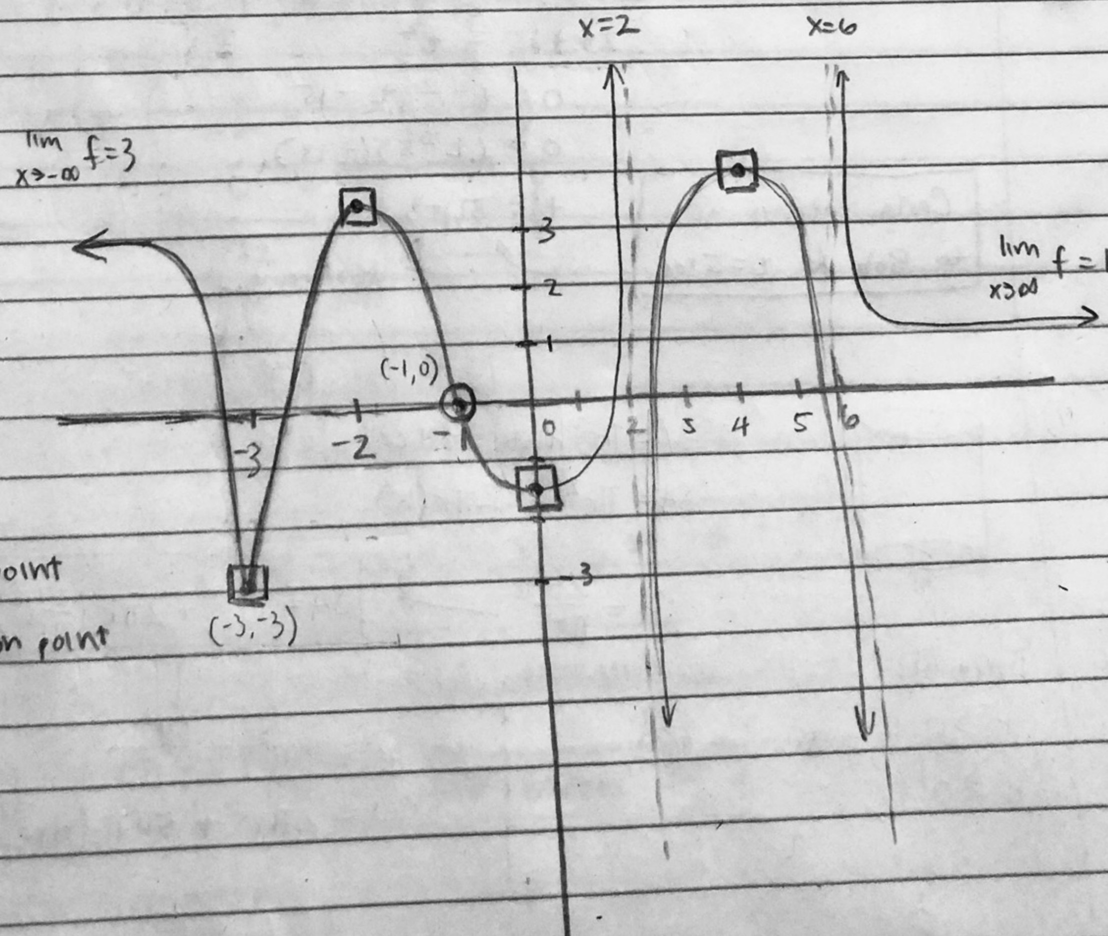
SA' − r = 3 +

Since SA' changes signs from − to +, r=3 is a min, not a max

CONCLUDE: $r=3$, $h = \frac{27\pi}{\pi r^2} = \frac{27}{9} = 3$

$\boxed{r=3,\ h=3}$

8) a)



□ = critical point

○ = inflection point