# 19F-MATH31A-1 Midterm 1

SAMUEL ALSUP

TOTAL POINTS

**96 / 100**

QUESTION 1

1 True and False **27 / 30**

✓ **- 3 pts** **Problem 6 Incorrect (F)**

QUESTION 2

2 Squeeze Theorem **15 / 15**

✓ **- 0 pts** **Correct**

QUESTION 3

3 Limit **7 / 7**

✓ **- 0 pts** **Correct**

QUESTION 4

Derivatives 18 pts

4.1 Limit Definition **3 / 3**

✓ **- 0 pts** **Correct**

4.2 Find Derivative **15 / 15**

✓ **- 0 pts** **Correct**

QUESTION 5

Intermediate Value Theorem 10 pts

5.1 IVT 1 **5 / 5**

✓ **- 0 pts** **Correct.**

5.2 IVT 2 **4 / 5**

✓ **- 1 pts** **The 1st condition is not satisfied: the domain of g is incorrect.**

QUESTION 6

Product and Quotient Rule 20 pts

6.1 Product Rule **10 / 10**

✓ **- 0 pts** **Correct**

6.2 Quotient Rule **10 / 10**

✓ **- 0 pts** **Correct**

# Midterm 1
# Math 31A-1, Fall 2019

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|---|---|

Please circle your section:

| Session 1A | Section 1B | Section 1C | Section 1D |
|---|---|---|---|
| Tuesday | Thursday | Tuesday | Thursday |
| Ben Erza Thompson | Ben Erza Thompson | Jason Brown | Jason Brown |

## Directions—Please read carefully!

- You are allowed **50 minutes** for this exam. Pace yourself, and do not spend too much time on any one problem.
- No notes, books, your own scratch papers, calculators, cell phones, computers, or other electronic aids are allowed.
- In order to receive full credit, you must **show your work or explain your reasoning**; your final answer is less important than the reasoning you used to reach it. Correct answers without work will receive little or no credit.
- Unless otherwise indicated, please simplify your answers.
- You can use the backs of pages as scratch papers, but **only those written in the front of pages** will be graded.
- Please write neatly. Illegible answers will be assumed to be incorrect. **Circle or box your final answer** when relevant.

Good luck!

| Question | Points | Score |
|---|---|---|
| 1 | 30 | |
| 2 | 15 | |
| 3 | 7 | |
| 4 | 18 | |
| 5 | 10 | |
| 6 | 20 | |
| Total: | 100 | |

(30) 1. Are the following statements True or False? NO explanation is needed for your answers.

True If $f(x)$ and $g(x)$ are continuous at $x = c$, then $\lim_{x\to c}[f(x) + g(x)] = f(c) + g(c)$.

True If $f(x)$ has a jump discontinuity at $x = a$, then $f(x)$ is not differentiable at $x = a$.

False If $f(x)$ is increasing near $x = a$, then $f'(a) > 0$.

False Because $\lim_{x\to 0} \sin x = 0$, we must have $\lim_{x\to 0} \frac{\sin x}{x} = 0$.

True If $f(x)$ and $g(x)$ are differentiable and $g(x) \neq 0$, then $\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$.

True Let $f(x)$ be a continuous function on the closed interval $[a, b]$ and $f(a) \geq 0, f(b) \leq 0$. Then there exists $c$ in the open interval $(a, b)$ such that $f(c) = 0$.

False The function $\tan x$ is differentiable at $x = a$ for every real number $a$.

False If $\lim_{x\to 0} f(x)$ exists while $\lim_{x\to 0} g(x)$ does not exist, then $\lim_{x\to 0} f(x)g(x)$ must not exist.

True If $f(x)$ and $g(x)$ are differentiable at $x = 0$ with $f'(0) = g'(0) = 0$, then $(f\cdot g)'(0) = 0$.

True Let $f(x)$ be a function defined on $(-\infty, \infty)$. Then there can be at most two horizontal asymptotes for $y = f(x)$.

(15) 2. Use the Squeeze Theorem to find $\lim_{x\to\infty} \frac{x}{x^2+1} \cos x$.

$\frac{x}{x^2+1} \cos x$

cos x has amplitude of 1

So we can squeeze cos x with -1 and 1

$\lim_{x\to\infty} \left|\frac{x}{x^2+1}\right|$ $\frac{\infty}{\infty}$ $\frac{1}{x^2} = \frac{1}{\infty} = 0$

$\lim |\cos x| \leq 1$

multiply by $\frac{x}{x^2+1}$

Now we take away absolute value sign and add negative side

$\lim_{x\to\infty} \left|\frac{x}{x^2+1} \cos x\right| \leq \left|\frac{x}{x^2+1}\right|$

$\lim_{x\to\infty} -\left|\frac{x}{x^2+1}\right|$ $\frac{\infty}{\infty}$ simplify to $-\frac{1}{x} = \frac{-1}{\infty} = 0$

$-\left|\frac{x}{x^2+1}\right| \leq \frac{x}{x^2+1} \cos x \leq \left|\frac{x}{x^2+1}\right|$

Since $\lim_{x\to\infty} -\left|\frac{x}{x^2+1}\right| = 0$

and $\lim_{x\to\infty} \left|\frac{x}{x^2+1}\right| = 0$ and

they squeeze $\frac{x}{x^2+1} \cos x$

$\boxed{\lim_{x\to\infty} \frac{x}{x^2+1} \cos x = 0}$

(7) 3. Let $f(x) = 3x - 6$ and $g(x) = x^2 - 8x + 1$. What is $\lim_{x\to 2} g(f(x))$?

$\lim_{x\to 2} g(f(x))$ Plug in $f(x)$ into $x$ for $g(x)$

$g(x) = (3x-6)^2 - 8(3x-6)+1$

Plug in 2

$g(2) = (3(2)-6)^2 - 8(3(2)-6)+1 =$

4. Let $f(x)$ be a function. $0 - 0 + 1 = \boxed{1}$

(3) (a) Write down the limit definition of the derivative $f'(x)$:

$$f'(x) = \boxed{\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}}.$$

(15) (b) Let $f(x) = \frac{1}{\sqrt{x}}$. Use the limit definition to find $f'(x)$.

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} \rightarrow \frac{\frac{\sqrt{x}}{\sqrt{x+h}\sqrt{x}} - \frac{\sqrt{x+h}}{\sqrt{x+h}\sqrt{x}}}{h}$$

Multiply to get same denominator

Multiply by conjugate

$$\frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x+h}\sqrt{x}} \cdot \frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x+h}+\sqrt{x}} = \frac{\frac{x-(x+h)}{\sqrt{x+h}\sqrt{x}(\sqrt{x+h}+\sqrt{x})}}{h}$$

Now, multiply to get rid of roots

Simplify to get

$$\frac{x-x-h}{\sqrt{x+h}\sqrt{x}(\sqrt{x+h}+\sqrt{x})} \over h$$

$$\frac{\frac{-\cancel{h}}{\sqrt{x+h}\sqrt{x}(\sqrt{x+h}+\sqrt{x})}}{\cancel{h}}$$

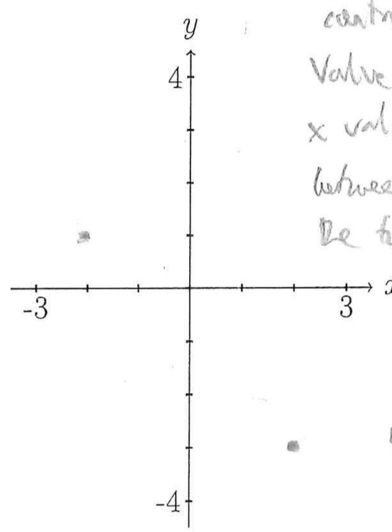
$$\lim_{h\to 0} \frac{-1}{\sqrt{x+h}\sqrt{x}(\sqrt{x+h}+\sqrt{x})}$$

Since $h \to 0$, simplify to

$$f' = \frac{-1}{\sqrt{x}\sqrt{x}(\sqrt{x}+\sqrt{x})} \rightarrow f'(x) = \frac{-1}{x\,2\sqrt{x}} \text{ or } \boxed{\frac{-1}{2x\sqrt{x}}} \text{ or } \boxed{2x^{-3/2}}$$

5. For each list of criteria, draw a graph satisfying all the criteria if it is possible to do so. **If it is impossible, explain why.**

(5) (a)
- The domain of $f(x)$ is the interval $(-3, 3)$
- $f(x)$ is continuous
- $f(-2) = 1$
- $f(2) = -3$
- There are no $x$ for which $f(x) = 0$



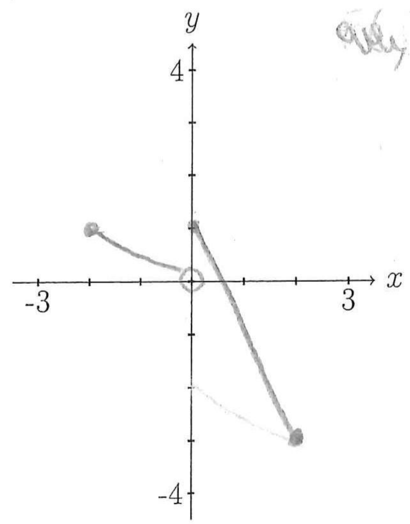

This is impossible, because on a continuous function, the Intermediate Value theorem guarantees that between two x values, every f(x) value will be touched between the two f(x) values found at the two x endpoints. In this case, f(x) = 0 will be crossed since f(-2) = 1 and f(2) = -3 which have opposite signs.

(5) (b)
- The domain of $g(x)$ is the interval $(-3, 3)$ ✓
- $g(x)$ is continuous except for a jump discontinuity at $x = 0$. ✓
- $g(-2) = 1$ ✓
- $g(2) = -3$ ✓
- For all $M$ in $[-3, 1]$, there exists $c$ in $[-2, 2]$ such that $g(c) = M$ ✓

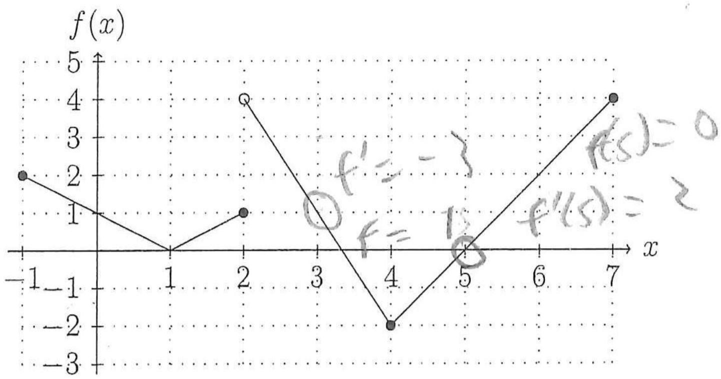
Possible

f(x) value

every y value in [-3, 1] is also hit between [-2, 2]



6. We consider two functions $f$ and $g$. Below is the graph of $f$ and a table of values for $g$.




| $x$ | $g(x)$ | $g'(x)$ |
|---|---|---|
| 0 | 4 | $-1$ |
| 1 | 3 | 0 |
| 2 | 1 | $\frac{1}{2}$ |
| 3 | 2 | 3 |
| 4 | 5 | $-1$ |
| 5 | 3 | $-6$ |

(10) (a) Find $\frac{d}{dx}[f(x) \cdot g(x)]$ at $x = 3$.

$\frac{d}{dx}(f(x) \cdot g(x))$ Use product rule : $f'(x) \cdot g(x) + f(x) \cdot g'(x)$

Plug in 3

$f'(3) \cdot g(3) + f(3) \cdot g'(3)$ ← Plug in values

$(-3 \cdot 2) + (1 \cdot 3) =$

$-6 + 3 = \boxed{-3}$

(10) (b) Find $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$ at $x = 5$.

$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$ Use quotient rule : $\frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$

$\frac{f'(5) \cdot g(5) - f(5) \cdot g'(5)}{(g(5))^2}$ Plug in values

$\frac{(2 \cdot 3) - (0 \cdot -6)}{3^2}$

$\frac{6 - 0}{9} = \frac{6}{9} = \boxed{\frac{2}{3}}$