Math 31A
Differential and Integral Calculus

Midterm 1

Instructions: You have 50 minutes to complete the exam. There are four problems, worth a total of 45 points. This test is closed book and closed notes. No calculator is allowed for this test. For full credit show all of your work legibly. Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name and UID in the space below.

Name: TYLER CARSON
Student ID number: 304.675.148

Question	Points	Score
1	10	10
2	10	10
3	15	14
4	10	10
Total:	45	44

Problem 1. *10pts.*

Suppose you are told that $y = 3x - 5$ is the tangent line to $f(x)$ at $x = 2$.

What is the line tangent to the function $g(x) = (x^2 + 1)f(x) - x$ at $x = 2$?

Be careful of making arithmetic errors.

If you do, but you demonstrate your method clearly enough, less points will be deducted.

$f(2) = 1$

$f'(2) = 3$

$g(x) = (x^2+1) f(x) - x$

if $h(x) = (x^2+1)$,

$h'(x) = 2x$

$g'(x) = h(x)f'(x) + f(x)h'(x) - \frac{d}{dx}(x)$

$g'(2) = (2^2+1)(3) + (1)(2(2)) - (1)$

$(5)(3) + (1)(4) - 1$

$15 + 4 - 1$

$g'(2) = 18$

$g(2) = (2^2+1)(1) - 2$

$5(1) - 2$

$g(2) = 3$

$f(2) = 1$
$f'(2) = 3$
$g(2) = 3$
$g'(2) = 18$

Tangent line @ $g(2)$ = $g(2) + g'(2)(x-2)$

$= 3 + 18(x-2)$

$= 18x - 36 + 3$

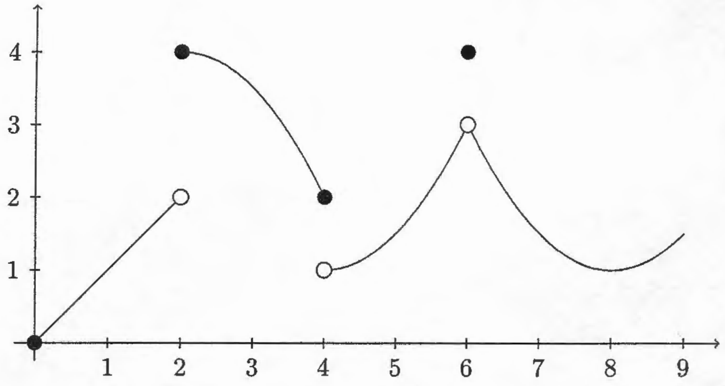
$= 18x - 33$

$\boxed{\text{@ } g(2), \text{ tangent line of } g(x) \text{ is } y = 18x - 33}$

10

Problem 2.

Let $f(x)$ be the function shown below.



Answer the following questions. (DNE is a possible answer.)

(a) [2pts.] $f'(1) =$ 1

(b) [2pts.] $\lim_{x \to 6} f(x) =$ 3

(c) [2pts.] Is $f(x)$ continuous at $x = 6$? If not, name the discontinuity. NOT CONTINUOUS: REPLACEMENT DISCONTINUITY

(d) [2pts.] Is $f(x)$ left continuous, right continuous or neither at $x = 2$? RIGHT CONTINUOUS

(e) [2pts.] $\lim_{x \to 2} f(x) f(x+2)$ 4

$\lim_{x \to 2+} f(x) = 2$

$\lim_{x \to 2-} f(x) = 4$

$\lim_{x \to 2+} f(x+2) = 2$

$\lim_{x \to 2-} f(x+2) = 1$

$\lim_{x \to 2+} f(x) f(x+2) = (2 \cdot 2) = 4$

$\lim_{x \to 2-} f(x) f(x+2) = (4 \cdot 1) = 4$

$$\boxed{\lim_{x \to 2} f(x) f(x+2) = 4}$$

Problem 3.

Evaluate the following limits.

(a) [5pts.] $\lim_{x\to 0}\left[(1-\cos x)\sin\left(\frac{1}{x}\right)\right]$

$$-1 \le \sin\left(\frac{1}{x}\right) \le 1$$

$$-1(1-\cos x) \le \sin\left(\frac{1}{x}\right)(1-\cos x) \le 1(1-\cos x)$$

$$\lim_{x\to 0} 1-\cos x = 0$$ ✓

$$-1(0) \le \sin\left(\frac{1}{x}\right)(1-\cos x) \le 1(0)$$

$$0 \le \sin\left(\frac{1}{x}\right)(1-\cos x) \le 0$$

$$\boxed{\lim_{x\to 0} (1-\cos x)\sin\left(\frac{1}{x}\right) = 0}$$ ✓

by Squeeze

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(b) [5pts.] $\lim_{x\to 3} \frac{x-\sqrt{x+6}}{x-3}$.

$$\lim_{x\to 3} \frac{x-\sqrt{x+6}}{x-3} =$$

$$= \frac{-1(x-\sqrt{x+6})}{-1(x-3)} = \frac{-x+\sqrt{x+6}}{3-x} = \frac{\sqrt{x+6}-x}{3-x} \frac{(\sqrt{x+6}+x)}{(\sqrt{x+6}+x)}$$

$$x^2+2x-3x-6 \checkmark$$

$$= \frac{(x+6)-x^2}{((\sqrt{x+6})+x)(3-x)} = \frac{(-x^2+x+6)(-1)}{(3-x)(\sqrt{x+6}+x)} = \frac{\cancel{(x+3)}(x-2)}{\cancel{(-x+3)}\sqrt{x+6}+x}$$

$$= \frac{-x-2}{\sqrt{x+6}+x} = \frac{-3-2}{\sqrt{9}+3} = \boxed{\frac{-5}{6}}$$

$$\boxed{\lim_{x\to 3} \frac{x-\sqrt{x+6}}{x-3} = \frac{-5}{6}}$$

Arithmetic error

4

$\sin^2 \cos^2 = 1$

(c) [5pts.] $\lim_{x\to 0} \frac{1-\cos x}{\sin x}$.

$$\lim_{x \to 0} \frac{1-\cos x}{\sin x} = \frac{1-\cos x\,(1+\cos x)}{\sin x\,(1+\cos x)} = \frac{1-\cos^2 x}{\sin(x)(1+\cos(x))}$$

$$= \frac{\sin^2 x}{\sin x\,(1+\cos x)} = \frac{\sin x}{1+\cos x} = \frac{\sin(0)}{1+\cos(0)} = \frac{0}{2} = 0$$

$$\boxed{\lim_{x \to 0} \frac{1-\cos x}{\sin x} = 0}$$

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Problem 4.

In each case calculate $f'(x)$

(a) [5pts.] $f(x) = (6x^4 - 3x^2 + 1)\sqrt{x}$

(b) [5pts.] $f(x) = \frac{x^4}{x^2+x+1}$

10

(a) $f(x) = (6x^4 - 3x^2 + 1)(x^{\frac{1}{2}})$

$(6x^4 - 3x^2 + 1)(\frac{1}{2}x^{-\frac{1}{2}}) + (24x^3 - 6x)(x^{\frac{1}{2}})$

$3x^{\frac{7}{2}} - \frac{3}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{-\frac{1}{2}} + (24x^{\frac{5}{2}} - 6x^{\frac{1}{2}})$

$3x^{\frac{7}{2}} + 24x^{5/2} - \frac{3}{2}x^{3/2} - 6x^{1/2} + \frac{1}{2}x^{-1/2}$

$f(x) = (6x^4 - 3x^2 + 1)(x^{\frac{1}{2}})$

$= 6x^{\frac{9}{2}} - 3x^{5/2} + x^{\frac{1}{2}}$

$= \frac{54}{2}x^{7/2} - \frac{15}{2}x^{3/2} + \frac{1}{2}x^{-1/2}$

$\boxed{27x^{7/2} - \frac{15}{2}x^{3/2} + \frac{1}{2}x^{-1/2}}$ ✓

(b) $\frac{x^4}{x^2+x+1} = \frac{(x^2+x+1)(4x^3) - (x^4)(2x+1)}{(x^2+x+1)^2}$

$= \frac{(4x^5 + 4x^4 + 4x^3) - (2x^5 + x^4)}{(x^2+x+1)^2}$

$= \boxed{\frac{2x^5 + 3x^4 + 4x^3}{(x^2+x+1)^2}}$ ✓