$\binom{7}{3} \times \frac{4!}{2!}$ Z O U

1. a)(10 pt) How many arrangements of the letters of MERCURY are there with the vowels appearing in alphabetical order (e.g., RECURMY).

b)(10 pt) How many arrangements of the letters in INSTRUCTOR have: the vowels and consonants appearing ~~in alphabetical order~~ and have at least 2 consonants between each vowel?

ⓐ Since for MERCURY, Y, E and U, are vowels,

so first chose portion for Y, E, U $= \binom{7}{3}$

_ E _ U _ Y _ Then randomly assign the

rest. $\frac{4!}{2!}$ ∴ # total $= \binom{7}{3} \times \frac{4!}{2!}$ ✓

ⓑ. for INSTRUCTOR., ~~vowels are I, U, O.~~

~~first assign positions for vowels $\binom{10}{3}$~~

_ I _ O _ U _ Then assign the rest

b_1 b_2 b_3 b_4

if you chose positions of I, U, O maybe there is no space left for 2 consonants

b_2 b_3 have at least two., we place four in $b_2 b_3$,

C N RR S T 3 ways to chose middle four.

Then we assign the remaining $\binom{10-3-4+4-1}{10-3-4}$

Thus # total $= \binom{6}{3} \times \binom{10-3-4+4-1}{10-3-4} \times 3$

0

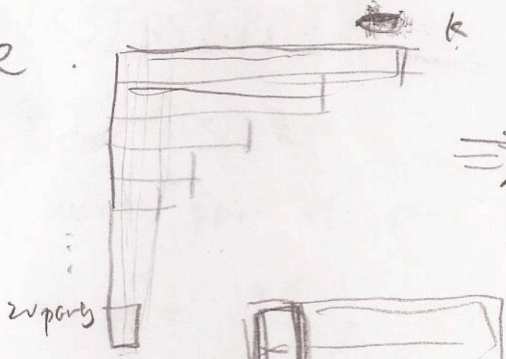
2 T.

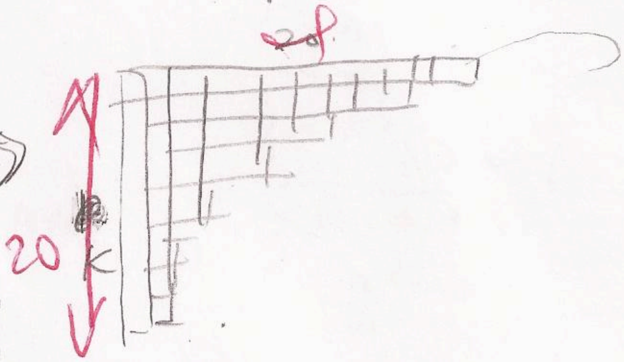
N S T 2 R I C

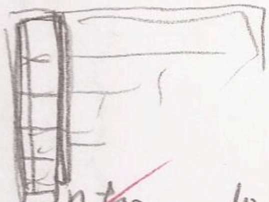
10 **2.**(10 pt) Show that the number of partitions of 30 into 20 parts is the same as number of partition of 20 into 10 parts.

since # of partition 30 into 20 parts.

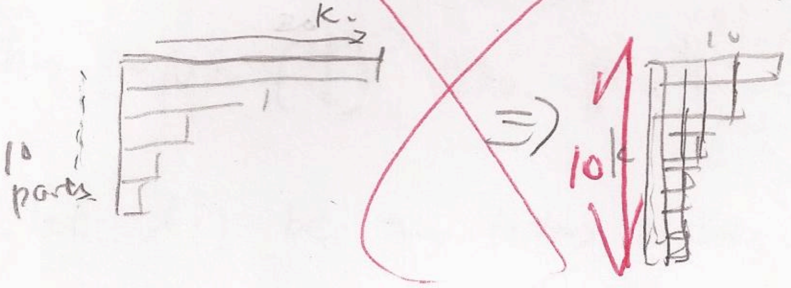
we here .







and partiam 20 into 10 parse.



2

you not explaining how to get one from another

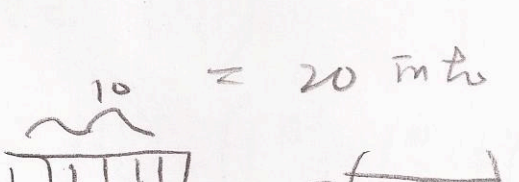
let k in case II be 20 case I be 10.

Also you draw diagrams incorrectly

Then # of 30 into 20

= 20 into 10

since



20 into 10

3. a)(10 pt) What is the probability that a pair of letters selected randomly from word MISSISSIPPI contains same letter twice? b)(15 pt) What is the probability that a triple of letters selected randomly from this word has all different letters.

ⓐ. for MISSISSIPPI.

1M 4I 4S 2P

Since we want pair of same letter

case I $2I \Rightarrow \# = \binom{4}{2}$

case II $2S \Rightarrow \# = \binom{4}{2}$

case III $2P \Rightarrow \# = \binom{2}{2}$

Thus $N = \binom{14}{2}$ Thus $P = \dfrac{\binom{4}{2}+\binom{4}{2}+\binom{2}{2}}{\binom{14}{2}}$

12

ⓑ. let A_i be two letters have same

$A_1 \cap A_2 = \binom{4}{2}+\binom{4}{2}+\binom{2}{2}$

$A_2 \cap A_3 = \binom{4}{2}+\binom{4}{2}+\binom{2}{2}$

$A_1 \cap A_2 \cap A_3 = (4\times3\times2)\times2 = \binom{4}{3}\times2$

and $N = \binom{11}{3}$

$\therefore \# = \binom{11}{3} - \left(\binom{4}{2}+\binom{4}{2}+\binom{2}{2}\right)\times2 + \binom{4}{3}\times2$

and $P = \dfrac{\cancel{\binom{11}{3} - \left(\binom{4}{2}+\binom{4}{2}+\binom{2}{2}\right)\times2 + \binom{4}{3}\times2}}{\binom{11}{3}}$

two letters S

$\binom{4}{2}\cdot 7$

Two letters I

$\binom{4}{2}\cdot 7$

Two letters P

$\binom{2}{2}\cdot 9$

3 letters

I, S $\binom{4}{3}+\binom{4}{3}$

4. You want to calculate the number of strings of length r you can make with digits 1, 2 and 3. The digits are available in infinite supply, but you must use at least one digit 2 and use either zero or at least two digits 3.

a)(15 pt) What generating function should you use for this problem? Is your function ordinary or exponential?

b)(10 pt) How many such words are there with $r = 10$ letters? You **must** use the generating function to calculate your answer.

(a). exponential function should be used since order matters,

$$g(x) = \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right)\left(x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right)\left(1 + \frac{x^2}{2!} + \frac{x^3}{3!} \cdots\right)$$

$\uparrow$ 1 $\qquad$ $\uparrow$ 2 $\qquad$ $\uparrow$ 3.

(b). $g(x) = e^x(e^x - 1)(e^x - x)$

$= (e^{2x} - e^x)(e^x - x)$

$= e^{3x} - xe^{2x} - e^{2x} + xe^x$

for $r = 10$ $\qquad$ $e_1 + e_2 + e_3 = 10$

we have coef $= \boxed{3^{10} - \frac{2^9 \cdot 10!}{9!} - 2^{10} + \frac{10!}{9!}}$

5.(20 pt) How many eight digit numbers (number can start with 0) are there with no digit appearing exactly three times.

Since any digits can be put in the buckets

We let A_i be digit i appear three times

$A_i = \binom{8}{3}$ 9^5

$A_i \cap A_j = \binom{8}{6} \frac{6!}{3!3!}$ 8^2

you can arrange remaining terms

$A_i \cap A_j A_k = 0$ since 9 exceede 8.

Thus $N = 10^8$

$\therefore N(\overline{A_1 A_2 \cdots A_{10}}) = (10^8) - \sum A_i + \sum A_{ij}$

$= \boxed{10^8 - \binom{8}{3} \times 10 + \binom{8}{6} \frac{6!}{3!3!} \times \binom{10}{2}}$

9^5

8^2

any two digit can appear 3 time each

9