20W-MATH170E-1 Midterm 1

DEBORAH LIU

TOTAL POINTS

47 / 50

QUESTION 1

1 1.(a) **5 / 5**

✓ **- 0 pts** Correct

- 1 pts Incorrect Probability Plug-in

- 2 pts Took incorrect complement

- 3 pts Incorrect events for "at least one"

- 3 pts Misusing Independence for Unions instead of Intersections

- 5 pts Did not attempt/ no serious progress toward solution

- 3 pts Did not use independence

QUESTION 2

2 1.(b) **5 / 5**

✓ **- 0 pts** Correct

- 0 pts Insignificant Arithmetic Error

- 2 pts Small error in formula for "Exactly one of the three occurs"

- 5 pts Severely Incorrect Formula for "Exactly one of the three occurs"

QUESTION 3

3 2.(a) **5 / 5**

✓ **- 0 pts** Correct

- 0 pts Correct answer assuming order matters in hands

- 1 pts Undercount by factor of 2

- 3 pts Missing some Binomial coefficients

- 2 pts Incorrect numbers in binomial coefficients

- 5 pts Severely Incorrect approach

- 3 pts Valid counting for an invalid notion of "hand"

QUESTION 4

4 2.(b) **2 / 5**

- 0 pts Correct

- 2 pts Correct Answer if order matters for hands (an easier problem)

✓ **- 3 pts** **Overcount by factor of 6**

- 3 pts Missing some binomial coefficients

- 5 pts Severely Incorrect Counting

- 2 pts Does not count EXACTLY one pair

- 2 pts Undercount by factor of 16

QUESTION 5

5 3 **10 / 10**

✓ **- 0 pts** Correct

- 4 pts Incorrect or Missing Application of Multiplication Rule

- 3 pts Incorrect Use of Total Probability

- 2 pts Failure to take Complements

- 2 pts Incorrect Bayes Theorem

- 1 pts Incorrect Numbers plugged in

- 10 pts Severely Incorrect Answer

QUESTION 6

6 4.(a) **5 / 5**

✓ **- 0 pts** Correct

- 2 pts Incorrect formula for the variance

- 4 pts Incomplete answer

QUESTION 7

7 4.(b) **5 / 5**

✓ **- 0 pts** Correct

- 2 pts Missing the value of PMF at 0

- 4 pts Incomplete answer

QUESTION 8

8 5.(a) **5 / 5**

✓ **+ 2 pts** **Know Poisson distribution**

✓ **+ 1.5 pts** **Correct set-up using the assumption**

✓ **+ 1.5 pts** **Correct answer**

+ **0 pts** Blank

QUESTION 9

9 5.(b) 5 / 5

✓ + **3 pts** **Know the correct mean for the # of accidents over the course two days. (2λ = 4)**

+ **3 pts** Know that the # of accidents over the course of two days is the sum of two independent Poisson RVs, corresponding to each of two days.

✓ + **2 pts** **Correct answer**

+ **1 pts** Some efforts have been made

+ **0 pts** (Essentially) Blank

Midterm 1

Name: Deborah Liu

Student ID: 2 0 5 1 4 0 7 2 5

Instructions:

- Do not open this exam until instructed to do so.
- You have 50 minutes to complete the exam.
- Please print your name and student ID number above.
- **You may not use calculators**, books, notes, or any other material to help you. Please make sure your **phone is silenced and stowed** where you cannot see it.
- Please write your solutions in the space below the problems. We will only grade your work within the pages that are relevant to the problems.
- Each problem is worth the same amount of points. Partial credit will be scarce, so make sure to get everything right.

Please do not write below this line.

Question	Score
1	
2	
3	
4	
5	
Total	

PROBLEM 1

Suppose that A, B, and C are mutually independent events such that $P(A) = 0.6$, $P(B) = 0.5$, and $P(C) = 0.2$.

(a) Compute the probability that at least one of the three events occurs.

(b) Compute the probability that exactly one of the three events occurs.

(a) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

$= 0.6 + 0.5 + 0.2 - 0.6 \times 0.5 - 0.5 \times 0.2 - 0.6 \times 0.2 + 0.6 \times 0.5 \times 0.2$

$= 1.3 - 0.3 - 0.1 - 0.12 + 0.06.$

$= 1.3 - 0.46.$

$= \cancel{0.54}\ 0.84.$

(b) P(exactly one of the three events occurs)

$= P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C)$

$= 0.6 \times 0.5 \times 0.8 + 0.4 \times 0.5 \times 0.8 + 0.4 \times 0.5 \times 0.2$

$= 0.24 + 0.16 + 0.04.$

$= 0.44.$

PROBLEM 2

A poker hand is defined as drawing five cards at random without replacement from a (standard) deck of 52 playing cards. Find the probability of each of the following poker hands:

(a) Full house – three cards of one rank, and two cards of another rank.
(For instance, 2♠ 2♡ 2♣ Q♡ Q♢)

(b) One pair – two cards of one rank, and three cards of three other ranks.
(For instance, 2♠ 2♡ 4♡ 8♠ J♣)

If you proceed by way of counting, clearly identify the counting method you are using.

(a) Total possible combinations: $\binom{52}{5}$.

each rank has 4 cards. 13 different ranks.

possible full house: $\cancel{\binom{4}{3}\binom{4}{2}}$. $13 \cdot \binom{4}{3} \cdot 12\binom{4}{2}$.

~~probability~~ $P(\text{full house}) = \dfrac{13\binom{4}{3}\cdot 12\binom{4}{2}}{\binom{52}{5}}$.

(b) possible one pair combination:

$\cancel{13\binom{4}{2} \cdot 12 \cdot \binom{12}{1} \cdot 11\binom{11}{}}$

$13\binom{4}{2} \cdot 12 \cdot \binom{4}{1} \cdot 11\binom{4}{1} \cdot 10\binom{4}{1}$.

$P(\text{one pair}) = \dfrac{13\binom{4}{2} 12\binom{4}{1} 11\binom{4}{1} 10\binom{4}{1}}{\binom{52}{5}}$.

PROBLEM 3

A hospital obtains 40% of its flu vaccine from Company A, 50% from Company B, and 10% from Company C. It is known that 2% of the vials from A are ineffective, 5% from B are ineffective, and 10% from C are ineffective. The hospital tests ten vials from a randomly selected shipment. If all of the ten are effective, find the conditional probability of that shipment's having come from C.

40% A: $(98\%)^{10}$ effective; $1-(98\%)^{10}$ not all 10 effective

50% B: $(95\%)^{10}$ effective; $1-(95\%)^{10}$ not all 10 effective

10% C: $(90\%)^{10}$ all effective; $1-(90\%)^{10}$ not all effective.

$$P(\text{shipment from } C \mid \text{all 10 effective}) = \cancel{\frac{0.1\times(0.9)^{10}}{0.1\times(0.9)^{10}+0.5\times(0.95)^{10}+0.4\times(0.98)^{10}}}$$

$$= \frac{P(\text{all effective} \mid \text{from } C)}{P(\text{all effective} \mid \text{from } A)+P(\text{all effective} \mid \text{from } B)+P(\text{all effective} \mid \text{from } C)}$$

$$= \frac{0.1\times(0.9)^{10}}{0.4\times(0.98)^{10}+0.5\times(0.95)^{10}+0.1\times(0.9)^{10}}$$

PROBLEM 4

Consider the random variable X of which the moment generating function is given by

$$M(t) = \frac{1}{6}(e^{-2t} + 3 + 2e^{t/2}).$$

Do the following:

(a) Compute $\mathrm{Var}(X)$.

(b) Find the probability mass function of X.

(a) $Var(X) = E(X^2) - [E(X)]^2$

$\cancel{E(X) = \frac{d}{dt}(\ )}$ $\quad E(X) = \frac{d}{dt} M(t)\Big|_{t=0} = \frac{1}{6}\frac{d}{dt}(e^{-2t} + 3 + 2e^{\frac{t}{2}})\Big|_{t=0}$

$= \frac{1}{6}(-2e^{-2t} + 2\cdot\frac{1}{2}e^{\frac{t}{2}})\Big|_{t=0}$

$= \frac{1}{6}(-2+1) = \frac{1}{6}\cdot(-1) = -\frac{1}{6}.$

$E(X^2) = \frac{d^2}{dt^2}M(t)\Big|_{t=0} = \frac{1}{6}(4e^{-2t} + \frac{1}{2}e^{\frac{t}{2}})\Big|_{t=0} = \frac{1}{6}(4+\frac{1}{2}) = \frac{1}{6}\cdot\frac{9}{2} = \frac{3}{4}$

$Var(X) = \frac{3}{4} - (-\frac{1}{6})^2 = \frac{3}{4} - \frac{1}{36} = \frac{27}{36} - \frac{1}{36} = \frac{26}{36} = \frac{13}{18}$

(b) $M(t) = E(e^{tX}) = \sum_{x \in S} p(x) e^{tx}$

$P(X=-2) = \frac{1}{6} \qquad P(X=0) = \frac{1}{2} \qquad P(X=\frac{1}{2}) = \frac{1}{3}.$

probability mass function:

$$P(X=x) = \begin{cases} \frac{1}{6}, & x=-2 \\ \frac{1}{2}, & x=0 \\ \frac{1}{3}, & x=\frac{1}{2} \\ \cancel{0, \ x \neq -2, 0, \frac{1}{2}} \end{cases}, \quad \cancel{x \in \{-2, 0, \tfrac{1}{2}\}}.$$

$S = \{-2, 0, \frac{1}{2}\}.$

$$e^{-\lambda}\frac{\lambda^x}{x!}$$

PROBLEM 5

Suppose that the number of car accidents on a particular highway assumes an approximate Poisson process. From past experience, it is known that the days with one or two car accidents are four times more frequent than the days with no car accidents.

(a) Find the expected number of car accidents on a given day.

(b) Find the probability that there is one car accident over the course of two days.

(a) λ: expected # of accident in 1 day.

$$P(X=1) = e^{-\lambda}\frac{\lambda}{1!} = \lambda e^{-\lambda}$$

$$P(X=2) = e^{-\lambda}\frac{\lambda^2}{2!} = e^{-\lambda}\frac{\lambda^2}{2} = \frac{1}{2}\lambda^2 e^{-\lambda}$$

$$P(X=0) = e^{-\lambda}\frac{\lambda^0}{0!} = e^{-\lambda}$$

$$\lambda e^{-\lambda} + \frac{1}{2}\lambda^2 e^{-\lambda} = 4e^{-\lambda}$$

$$\lambda + \frac{1}{2}\lambda^2 = 4$$

$$\lambda^2 + 2\lambda - 8 = 0$$

$$(\lambda+4)(\lambda-2) = 0.$$

$$\lambda = 2, -4$$

$\because$ we can't have negative number of car accident $\therefore \boxed{\lambda = 2}$

(b) $\lambda' = 2\lambda = 4.$ λ': the expected number of car accident in 2 days.

$$P(X=1) = e^{-\lambda'}\cdot\frac{\lambda'}{1!} = \lambda' e^{-\lambda'} = 4e^{-4}.$$

$\cancel{2e^{-2}\cdot e^{-2}}$

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