Midterm 2, 170E, Spring 2021
Instructor: Kyeongsik Nam

Printed name: ______________________________

Signed name: ______________________________

Student ID number: ______________________________

Instructions

- Read problems very carefully. If you have any questions, send an email to the instructor.
- This is an open book, open notes, and open internet take-home exam.
- You have 24 hours to complete the exam between **May 10, Monday, 8AM - May 11, Tuesday, 8AM**. Submit the exam through the Gradescope.
- Justify everything you write as much as possible. There will be no partial credit for just guessing the correct final answer alone. Unless otherwise stated, directly citing past home-work problems or results in the lecture note and not showing your work will only get partial credit. Your solution should be mostly self-contained.

Question	Points	Score
1	15	
2	15	
3	20	
4	20	
5	30	
Total	100	

Please sign the following statement below, and print and sign your full name afterwords.

"I assert, on my honor, that I have not received assistance of any kind from any other person and that I have not used any non-permitted materials or technologies during the period of this evaluation."

Statement:

Print your full name:

Signature:

1. (15 points) Suppose that X and Y are independent Hypergeometric random variables with parameters $N_1 = 6$, $N_2 = 3$, and $n = 2$. Compute $P(X = Y)$.

2. (15 points) A continuous random variable X has a cumulative distribution function

$$F(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ x^2 & \text{if } 0 < x < 1 \\ 1 & \text{if } x \geq 1. \end{cases}$$

Compute the mean and variance of X.

3. (20 points) Suppose that arriving times of customers at the shop follows a Poisson process with mean 3 (per minute). Conditioned that no customers arrived at the shop for 25 minutes, compute the probability that no customers arrive additional 10 minutes or more.

4. (20 points) Let X be a normal distribution with mean 3 and variance 4. Find the probability density function of a random variable $Y = -2X - 1$.

5. (30 points) There are total 12 many balls in the box: 10 of them are red, 1 is green, and 1 is blue. Select the ball one at a time, without replacement, until the box is empty.

Let $X = 1$ if all of the red balls are selected, before the green ball is selected; and $X = 0$ otherwise. Let $Y = 1$ if all of the red balls are selected, before the blue ball is selected; and $Y = 0$ otherwise.

Find the covariance of X and Y.