

**Midterm 1, 170E, Spring 2021**  
**Instructor: Kyeongsik Nam**

Printed name: \_\_\_\_\_

Signed name: \_\_\_\_\_

Student ID number: \_\_\_\_\_

**Instructions**

- Read problems very carefully. If you have any questions, send an email to the instructor.
- This is an open book, open notes, and open internet take-home exam.
- You have 24 hours to complete the exam between **Apr 19, Monday, 8AM - Apr 20, Tuesday, 8AM**. Submit the exam through the Gradescope.
- Justify everything you write as much as possible. There will be no partial credit for just guessing the correct final answer alone. Unless otherwise stated, directly citing past home-work problems or results in the lecture note and not showing your work will only get partial credit. Your solution should be mostly self-contained.

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

**Please sign the following statement below, and print and sign your full name afterwords.**

*"I assert, on my honor, that I have not received assistance of any kind from any other person and that I have not used any non-permitted materials or technologies during the period of this evaluation."*

Statement:

Print your full name:

Signature:

1. (20 points) Suppose that events  $A$  and  $B$  satisfy

$$P(A) = 0.3, \quad P(B) = 0.4, \quad P(A \cap B) = 0.1.$$

Compute  $P(A'|B')$ . Here,  $A'$  and  $B'$  denote the complement of  $A$  and  $B$ , respectively.

2. (20 points) Two dice with numbers 1,2,3,4,5,6 on the face are rolled. Let  $A$  be the event that sum of two dice equals 5, and let  $B$  be the event that at least one of the dice shows 1. Are  $A$  and  $B$  independent?

3. (20 points) Suppose that we have  $k$  distinct balls (numbered 1 through  $k$ ) and  $n$  distinct boxes (numbered 1 through  $n$ ). We are asked to put away the balls by putting them into the boxes, each of the possible arrangements being equally likely. What is the probability that the first box (numbered 1) contains exactly three many balls? We assume that  $k \geq 3$ .

4. (20 points) Assume that 50% of emails are spam emails. There is a software detecting spam emails. The probability that a spam email detected as spam is 99%, and the probability that a non-spam email detected as non-spam is 95%. If an email is detected as spam, then what is the probability that it is in fact a spam email?

5. (20 points) Let  $X$  be a random variable having a Binomial distribution with  $n = 2$  and  $p = 0.2$ . Compute the expectation of  $X^{X+1}$ .