

170E: Midterm 2 - Fall 2020

This is an open-book exam, i.e. you may use (almost) all materials and sources. However, your solutions should be based on the material covered in this course.

Collaborations of any kind (also through online forums) are strictly prohibited.

See the file "Exam Administration" attached at the very end of this document for details.

Start time: November 23, 2020, 8am PST

End time: Tuesday, November 24, 2020, 8am PST.

How to take the exam: a step-by-step instruction

- Find yourself a calm place with good internet connection where you can work on the exam for 1-2 hours (or however much time you think you need) without distraction. Reserve enough time for the submission process.
- Copy the Academic Honesty Statement (next page) onto a blank page (handwritten either pen-on-paper or on tablet) and make sure your name and your UID are very legible. Do not forget to sign it.
- Start working on your exam. You may write on a tablet, on paper, or type in LaTeX (or similar). Make sure your solutions are complete and legible.
- If any problems occur while taking the exam: send me an email and allow up to 1-2 hours for an answer (potentially more during PST night). Do not post about it on any platform (also not CCLE).
- When you are done taking the exam: scan or photograph your solutions including the honesty statement. Compose all pages into one good-quality and reasonable-size pdf. Double check the document for completeness before you submit.
- Submit your file through Gradescope before Tuesday, November 24, 2020, 8am PST.

UCLA Math-170E: Commitment to academic honesty

I am aware of the UCLA Student Conduct Code, and specifically Section 102.01 on academic dishonesty.

I assert, on my honor, that I will not receive assistance of any kind from any other person while working on this exam.

I am aware that (i) deviation from this rule, or late submission of my work, will render my exam void; (ii) the instructor may contact me after the exam to ask for additional explanation of my answers; (iii) the instructor will report any suspected attempt of violation of student conduct code to the Dean of Students office.

NAME:

UID:

SIGNATURE:

DATE:

Copy this Academic Honesty Statement onto a blank page (handwritten: either pen-on-paper or on tablet) and make sure your name and your UID are legible, and make sure to sign it.

SHOW YOUR WORK IN ALL PROBLEMS!

This means in particular: indicate where you use the assumptions given, e.g. say “since the events are exclusive”, or “because the sum is finite”; name theorems that you are applying, e.g. say “by the partition theorem”, or “by the binomial formula”; write out intermediate steps in computations; give additional explanation in words if necessary. In particular, if your answer is not perfectly correct but your approach is comprehensible then you are likely to receive partial credit.

Whenever possible, compute your results without rounding (e.g. write simplified fractions instead of decimals). If that is not possible (or does not make sense in a particular problem), then round to four decimal places.

Total points: 50

1. (10 points) A random point (X, Y) is chosen at random from the set

$$S = \{(x, y) \in \mathbb{Z}^2 : -2 \leq x \leq 2 \text{ and } -3 \leq y \leq 3\}$$

so that each point in the set is chosen with equal probability.

- Draw a picture of the set S .
- Give the joint PMF of X and Y , and compute the marginal PMFs of X and Y .
(*Hint:* Give the full table for the joint and marginal PMFs in case this helps your understanding. However, it is not necessary and you are not required to.)
- Are $X + Y$ and $X - Y$ independent?
- Compute $\mathbb{E}[(X - Y)(X + Y)]$.

2. (6 points) A ball launcher on a baseball court shoots a ball at velocity 1 meter per second at a random angle $\theta \sim \text{Uniform}([\frac{\pi}{6}, \frac{\pi}{4}])$. The distance D in meters travelled by the ball before it lands is given by

$$D = \frac{1}{10} \sin(2\theta).$$

What is the expected distance the ball travels before it lands?

Show your working and give the results without having to round if possible (e.g. by writing it in a simplified fraction). If that is not possible, then round to four decimal places.

3. (6 points) Let $X \sim \text{Uniform}(-1, 1)$ and $Y = e^{X^2}$. Find the PDF of Y .

4. (6 points) Let X, Y be random variables so that

$$\mathbb{E}[X] = \frac{1}{2}, \quad \mathbb{E}[Y] = -1, \quad \mathbb{E}[X^2] = \frac{1}{2}, \quad \text{Var}(Y) = 4, \quad \mathbb{E}[XY] = \frac{1}{8}.$$

- (a) Find the correlation coefficient $\rho(X, Y)$.
- (b) Can you conclude from the given information, whether X and Y independent? Explain your answer in one or two sentences.
5. (10 points) People arrive with an average rate of 20 people per hour at border control. We assume that arrivals are modelled by a Poisson process
- (a) What is the probability that at least 3 people arrive within the first two hours?
- (b) What is the expected number of people arriving within the first two hours?
- (c) What is the probability that the first customer arrives within the first 30 minutes?
- (d) What is the expected arrival time of the 13-th customer?
- (e) (BONUS) Given that exactly two customers have arrived in the first 30 minutes. What is the expected arrival time of the third customer? Explain your answer.

Show your working in all parts and indicate when you are using the fact that a certain random variable has certain type of distribution.

6. (6 points) Suppose that $X \sim N(70, 16)$. What is the probability that $X \leq 64$?
- (a) Find the solution by using the table for the “Standard Normal Distribution Function” in the book. Write out all computational steps that you take in order to use the table and write your result for this part without rounding.
- (b) Find the solution by using a computational resource such as your calculator or a math program. Write down which resource you are using what you typed in order to obtain the answer (a screen-shot works too). Give the result rounded to four decimal places.
7. (6 points) A shipment of 8 pairs of custom-made earrings is on its way to a jewellery store. Each pair is unique. During the transport 10 of these 16 earrings are stolen. What is the expected number of complete pairs of earrings that make it to the store?
- (*Hint*: for each of the 8 pairs, consider the random variable that is equal to 1 if the pair makes it to the store, and is equal to 0 if one or both of the earrings of the pair were stolen. How is the number of complete pairs connected to these random variables?)

Exam Administration (for all exams with instructor A. Iseli in Fall 2020)

Exam Format and Preparation

- The midterms are designed as 1 hour exams, the final is designed as a 3 hour exam. This information is just to give you an estimate on how much time out of your day you should reserve for working on the exam. However, all exams are given **24 hours for completion** (by departmental policy) and you may work on your solutions for as long as you wish to within these 24 hours.
- Do not let this generous time window mislead you - prepare and study for the exam similar as you would in usual times. In addition, spend some thought on **time management** prior to the exam. In particular, reserve enough time within your productive hours of the day to take the exam; plan on where you take it (a calm place where you can focus); and plan enough time for scanning and submitting your solutions (schedule a little time-buffer in case you have to resolve a technological issue last minute).

How to take an exam: a step-by-step instruction

- Download the exam from [CCLE](#). (The exam is available from 8am of the respective day. Dates of exams can be found in the syllabus of the respective course page.)
- Carefully read the instruction on the first page of the exam
- Copy the Academic Honesty Statement (will be included in exam file) onto a blank page (handwritten pen-on-paper or tablet) and make sure your name and your UID are very legible.
- If any problems occur while taking the exam: send me an email and allow up to 1-2 hours for an answer (potentially more during PST night). Do not post about it on any platform (also not CCLE).
- When you are done taking the exam: scan or photograph your solutions as well as the honesty statement. Compose all pages into one good-quality and reasonable-size pdf. Double check the document for completeness before you submit. Then submit your file through [Gradescope](#).
- If something does not work smoothly (e.g. with the upload): please do not panic, take a deep breath, wait a minute or two, then try one more time. If it still does not work: still do not panic and email me as quickly as possible.

Rules and Guidelines

- The final is an **open book exam**: you are allowed to use almost all source available: books, lecture notes, homework, google, wikipedia, wolframalpha.com, existing posts on stackexchange, and other similar sources.
- **Prohibited sources** are: chegg.com, as well as posts that contain content (potentially posted by your peers) that violate the next bullet point. Please report such sources to me immediately.
- **Collaboration of any sort is strictly prohibited**. In particular, do not ask other people for help for hints or to solve the problems for you, don't share your solutions with others, do not post or discuss problems related to the exam on any sort of (online) platform.
- Your solutions should be based on **theory and techniques taught in this course**: applying theorems that were not covered in this course (and that you cannot easily prove using the techniques and theorems from this course) will result into little to zero credits for the respective problem.
- **Referencing is strongly encouraged**: e.g. if in a problem you are asked to come up with an example for something and you happen to find one on (say) math stack exchange, I prefer if you let me know about the source. This won't result in a loss of points for you, and it might actually prevent further questions from my side during the grading process (see next bullet).
- Be aware that the instructor has the right to ask for explanations of your solutions at any point of the grading process.
- Be reminded of the **Students Conduct Code (SCC)**. Amongst other things, it states that you follow rules and policies set by the department and the instructor, and to not violate academic honesty. The exam will contain a preprinted statement for you to sign that says that you assure that you are aware of the SCC and that you did not violate it while taking the final of this course.
(<https://www.deanofstudents.ucla.edu/Individual-Student-Code>)

Questions/Problems during the exam

- You may come across a problem while working on your exam: technical/computer issue, a suspected typo in an exam problem, an exam problem that needs clarification, an uncertainty about allowed resources, etc. In this case, please **contact me by email** and allow 1-2 hours for an answer (possibly more during PST night-time). I will try to be available as much as I can by email during the 24h time window.
- I will repost all relevant questions (e.g. on potential typos in the exam) that I receive by email together with my response on the **CCLE forum**. Do not post about your problem/concern/question by yourself on CCLE forum (nor other forums).