170E: Midterm 1 - Fall 2020

This is an open-book exam, i.e. you may use (almost) all materials and sources. However, your solutions should be based on the material covered in this course.

Collaborations of any kind (also through online forums) are strictly prohibited.

See the file "Exam Administration" attached at the very end of this document for details.

Start time: Monday, October 26, 2020, 8am PST

End time: Tuesday, October 27, 2020, 8am PST.

How to take the exam: a step-by-step instruction

- Find yourself a calm place with good internet connection where you can work on the exam for 1-2 hours (or however much time you think you need) without distraction. Reserve enough time for the submission process.
- Copy the Academic Honesty Statement (next page) onto a blank page (handwritten either pen-on-paper or on tablet) and make sure your name and your UID are very legible. Do not forget to sign it.
- Start working on your exam. You may write on a tablet, on paper, or type in LaTex (or similar). Make sure your solutions are complete and legible.
- If any problems occur while taking the exam: send me an email and allow up to 1-2 hours for an answer (potentially more during PST night). Do not post about it on any platform (also not CCLE).
- When you are done taking the exam: scan or photograph your solutions including the honesty statement. Compose all pages into one good-quality and reasonable-size pdf. Double check the document for completeness before you submit.
- Submit your file through Gradescope before Tuesday, October 27, 2020, 8am PST.

UCLA Math-170E: Commitment to academic honesty

I am aware of the UCLA Student Conduct Code, and specifically Section 102.01 on academic dishonesty.

I assert, on my honor, that I will not receive assistance of any kind from any other person while working on this exam.

I am aware that (i) deviation from this rule, or late submission of my work, will render my exam void; (ii) the instructor may contact me after the exam to ask for additional explanation of my answers; (iii) the instructor will report any suspected attempt of violation of student conduct code to the Dean of Students office.

NAME:

UID:

SIGNATURE:

DATE:

SHOW YOUR WORKING IN ALL PROBLEMS!

This means in particular: indicate where you use the assumptions given (e.g. say "since the events are exclusive" or "because the sum is finite"); name theorems that you are applying (e.g. say "by partition theorem" or "by the binomial formula"), write out intermediate steps in computations; give additional explanation in words if necessary. (In particular, if your answer is not perfectly correct but your approach is comprehensible then you are likely to receive partial credit.)

Total points: 50

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1. (8 points) You are organizing a party to which you invite 200 guests but only the first 120 arriving will be let in. The guests will arrive in random order. The door man lists the first names of the first 120 guests in the order they arrive.
   (a) Assume that all of the 200 potential guests have distinct names. How many different lists are possible?
   (b) Now assume that among the 200 potential guests there are 4 that are called Ajay, 5 are called Meiling, 51 are called Ana, and all other guests have distinct names. How many different lists are possible?
   (c) How many different lists are possible in Part (b) if we (unlike above) do NOT care about the order in which the first 120 guests arrive?

2. (8 points) Let $A, B, C \subseteq \Omega$ be (mutually) independent events so that
$$\mathbb{P}[\,A \cup B\,] = \frac{5}{8}, \qquad \mathbb{P}[\,A \cap B^c\,] = \frac{3}{8}, \qquad \mathbb{P}[\,A \cup B \cup C\,] = \frac{3}{4}.$$
   (a) What is $\mathbb{P}[\,B\,]$?
   (b) What is $\mathbb{P}[\,A\,]$?
   (c) What is $\mathbb{P}[\,C\,]$?

3. (BONUS) Let $A, B, C \subseteq \Omega$ be events in a probability space. Assume that A and B are independent; A and C are independent; C and B are exclusive. Prove that A and $B \cup C$ are independent.

4. (20 points) You missed class last week, so you don't know which chapters will be part of Monday's exam. The only way to find out is asking one of your 72 classmates. Among those, 20 are trustworthy people who will give you the correct answer for sure; 48 never pay attention in class and therefore will give you a correct answer with a probability of only 60%; and 4 are notorious liars and will give you the wrong answer for sure. You ask one class mate at random (each of the 72 class mate is asked with equal probability).
   (a) What is the probability that the class mate asked is not one of the trustworthy people?
   (b) If you ask one classmate at random, what is the probability that you will obtain a correct answer?
   (c) Are the two following events A and B independent?
   $A = \{\text{You obtain a correct answer}\}, \quad B = \{\text{You ask one of the notorious liars}\}$
   (Hint: check whether the definition of independent events applies.)
   (d) Are the events A and B from Part (b) exclusive?
   (Justify your answer in one or two sentences or by a formula.)
   (e) After the exam, you notice that you were given an incorrect answer. What is the probability that you had asked one of the notorious liars?
   (f) For this part, we alter the experiment a bit: You first ask one out of all 72 class mates at random, then you ask another class mate out of the remaining 71 at random. What is the probability that you obtain twice the same answer?

5. (8 points) It costs me \$1 to play a game. If I win I get \$2 and if I lose I get \$0. I have a probability $\frac{1}{4}$ of winning each game, independently of all other games. Let X be my profit in dollars after 20 games (which could be negative). Be aware that "profit" means winnings minus spendings.
   (a) Explain why the set of values of X is $S = \{-20, -18, -16, \ldots, 18, 20\}$.
   (b) Find the PMF of X.

6. (6 points) Define the function $f : \mathbb{N} \to \mathbb{R}$ as follows:

$$f(x) = \begin{cases} c, & \text{if } x = 0 \\ \frac{1}{2x^2} & \text{if } x \in \mathbb{N} \\ 0 & \text{if } x \notin \mathbb{N}_0 \end{cases}$$

   where $c > 0$ is a constant. For which value(s) of the constant c is the function f the PMF of a discrete random variable?

   (Hint: You may use (without proof) that: $\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$)

   In addition to doing the necessary computations, give a short explanation why your choice of constant c makes f the PMF of a discrete random variable.

Exam Administration (for all exams with instructor A. Iseli in Fall 2020)

Exam Format and Preparation

- The midterms are designed as 1 hour exams, the final is designed as a 3 hour exam. This information is just to give you an estimate on how much time out of your day you should reserve for working on the exam. However, all exams are given **24 hours for completion** (by departmental policy) and you may work on your solutions for as long as you wish to within these 24 hours.
- Do not let this generous time window mislead you - prepare and study for the exam similar as you would in usual times. In addition, spend some thought on **time management** prior to the exam. In particular, reserve enough time within your productive hours of the day to take the exam; plan on where you take it (a calm place where you can focus); and plan enough time for scanning and submitting your solutions (schedule a little time-buffer in case you have to resolve a technological issue last minute).

How to take an exam: a step-by-step instruction

- Download the exam from CCLE. (The exam is available from 8am of the respective day. Dates of exams can be found in the syllabus of the respective course page.)
- Carefully read the instruction on the first page of the exam
- Copy the Academic Honesty Statement (will be included in exam file) onto a blank page (handwritten pen-on-paper or tablet) and make sure your name and your UID are very legible.
- If any problems occur while taking the exam: send me an email and allow up to 1-2 hours for an answer (potentially more during PST night). Do not post about it on any platform (also not CCLE).
- When you are done taking the exam: scan or photograph your solutions as well as the honesty statement. Compose all pages into one good-quality and reasonable-size pdf. Double check the document for completeness before you submit. Then submit your file through Gradescope.
- If something does not work smoothly (e.g. with the upload): please do not panic, take a deep breath, wait a minute or two, then try one more time. If it still does not work: still do not panic and email me as quickly as possible.

Rules and Guidelines

- The final is an **open book exam**: you are allowed to use almost all source available: books, lecture notes, homework, google, wikipedia, wolframalpha.com, existing posts on stackexchange, and other similar sources.
- **Prohibited sources** are: chegg.com, as well as posts that contain content (potentially posted by your peers) that violate the next bullet point. Please report such sources to me immediately.
- **Collaboration of any sort is strictly prohibited**. In particular, do not ask other people for help for hints or to solve the problems for you, don't share your solutions with others, do not post or discuss problems related to the exam on any sort of (online) platform.
- Your solutions should be based on **theory and techniques taught in this course**: applying theorems that were not covered in this course (and that you cannot easily prove using the techniques and theorems from this course) will result into little to zero credits for the respective problem.
- **Referencing is strongly encouraged**: e.g. if in a problem you are asked to come up with an example for something and you happen to find one on (say) math stack exchange, I prefer if you let me know about the source. This won't result in a loss of points for you, and it might actually prevent further questions from my side during the grading process (see next bullet).
- Be aware that the instructor has the right to ask for explanations of your solutions at any point of the grading process.
- Be reminded of the **Students Conduct Code (SCC)**. Amongst other things, it states that you follow rules and policies set by the department and the instructor, and to not violate academic honesty. The exam will contain a preprinted statement for you to sign that says that you assure that you are aware of the SCC and that you did not violate it while taking the final of this course.
(`https://www.deanofstudents.ucla.edu/Individual-Student-Code`)

Questions/Problems during the exam

- You may come across a problem while working on your exam: technical/computer issue, a suspected typo in an exam problem, an exam problem that needs clarification, an uncertainty about allowed resources, etc. In this case, please **contact me by email** and allow 1-2 hours for an answer (possibly more during PST night-time). I will try to be available as much as I can by email during the 24h time window.
- I will repost all relevant questions (e.g. on potential typos in the exam) that I receive by email together with my response on the **CCLE forum**. Do not post about your problem/concern/question by yourself on CCLE forum (nor other forums).