3. (3 points) Let $X$ be a random variable with density

$$f(x) = \begin{cases} e^{-x}, & x \geq 1 \\ 1 - e^{-1}, & 0 \leq x < 1 \\ 0, & x < 0 \end{cases}.$$

Let $F_X$ be the cumulative distribution function of $X$. Then for $t \geq 1$, $F_X(t) =$

- A. $1 - e^{-t}$
- B. $e^{-1} - e^{-t}$
- C. $(1 - e^{-1})t$
- D. $e^{-1} + e^{-t}$
- E. $1 + e^{-t-1}$

$\int_0^1 (1-e^{-1})\,dx = 1-e^{-1}(x)\Big|_0^1$

$= 1-e^{-1}$

$\int_1^t e^{-x}\,dx = -e^{-x}\Big|_1^t$

$= -(e^{-t}-e^{-1})$

$= e^{-1}-e^{-t}$

$(1-e^{-1})+(e^{-1}-e^{-t})$

$1-e^{-t}$

4. (3 points) Suppose that $X$ and $Y$ have joint density

$$f(x, y) = c(xy + x^2y^2)I(x \in [0, 3] \text{ and } y \in [0, 1]).$$

Then $c =$

- A. 7/18
- B. 9/4
- C. 4/21
- D. 4/9
- E. 18/7

$\int_0^1 \int_0^3 (xy + x^2y^2)\,dx\,dy$

$\left[\frac{x^2y}{2} + \frac{x^3y^2}{3}\right]_0^3 = \frac{9y}{2} + 9y^2$

$\int_0^1 \left(\frac{9y}{2} + 9y^2\right)dy = \frac{9y^2}{4} + 3y^3\Big|_0^1$

$= \frac{9}{4} + 3 = \frac{9+12}{4} = \frac{21}{4}$

$c\left(\frac{21}{4}\right) = 1 \Rightarrow c = 4/21$

End of quiz

Total score: 12 points

Full name: Kanisha Shah (1 point)

12/12

1. (3 points) If $X \sim \text{Geom}(0.02)$ then $\mathbb{P}(50 \le X \le 150) \approx$
   - A. $e^{-0.5} - e^{-1.5}$
   - B. $e^{-1} - e^{-3}$
   - C. $e^{-0.5} - e^{-3}$
   - D. $e^{-2} - e^{-3}$
   - E. $e^{-1} - e^{-4}$

$\lambda e^{-\lambda t}$

$$\int_{50}^{150} \lambda e^{-\lambda t}\, dt = -e^{-\lambda t}\Big|_{t=50}^{t=150}$$

$$-\left[e^{-(150)(0.02)} - e^{-(0.02)(50)}\right]$$

$$= -\left[e^{-3} - e^{-1}\right] = e^{-1} - e^{-3}$$

$1 \le pX \le 3$

$$\int_1^3 e^{-t}\, dt = \frac{e^{-t}}{-1}\Big|_1^3$$

$-(e^{-3} - e^{-1})$

2. (2 points) Let $X$ be a random variable with density $f(x) = xe^x I(0 \le x \le 1)$. Find $\mathbb{E}e^{-X}$.
   - A. $1/e$
   - B. $1/2$
   - C. $1/(2e+1)$
   - D. $1/(2e)$
   - E. $1/(e+2)$

$$\int_0^1 x e^x e^{-x}\, dx = \frac{x^2}{2}\Big|_0^1 = \frac{1}{2}$$