

8. (6 points) A box contains 5 blue balls, 6 red balls and 7 purple balls. Three balls are drawn from the box without replacement. Write out the expression for (but no need to evaluate) the probability that the first ball is not blue, the second ball is red and the third ball is purple. No justification needed. Only the final answer worths credit.

3 Choices, multiplication principle. 18 total balls, no replace.

Choice 1: not blue.

$$\frac{6/18}{R} (5b, 6r, 7p) \quad \frac{7/18}{P}$$

Choice 2: Red

$$\frac{5/17}{R} (5b, 5r, 7p) \quad \frac{6/17}{R}$$

Choice 3: Purple

$$\frac{7/16}{P} (5b, 4r, 7p) \quad \frac{6/16}{P} (5b, 5r, 6p)$$

$$P(1b, 1r, 1p) = \left( \frac{6}{18} \cdot \frac{5}{17} \cdot \frac{7}{16} \right) + \left( \frac{7}{18} \cdot \frac{6}{17} \cdot \frac{6}{16} \right)$$

Question	Points Awarded	Points
Name		1
1-6		18
7		5
8		6
		30

End of exam

Part II: WORK OUT

7. (5 points) At a company, 75% of the workers have electricity license, 55% have gas pipe license and 40% have both license. What is the probability that a selected worker has either electricity or gas pipe license but not both? Show your work and box your final answer.

Looking for the symmetric difference. Let event  $E$  be the event where a worker has an electricity license +  $G$  be the event where a worker has a gas pipe license.



$P(E \cap G)$  is shaded.

Want to find  $P(E \Delta G) = P(E \cap G^c) + P(G \cap E^c)$

$P(E) = 0.75$

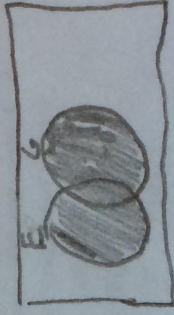
$P(E) + P(G) - P(E \cap G) = P(E \cup G)$

$P(G) = 0.55 \Rightarrow$

$1.3 - 0.4 = P(E \cup G) = 0.9$

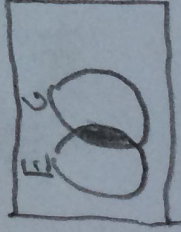
$P(E \cap G) = 0.4$

↳



$P(E \cup G)$  is shaded.

Note that  $P(E \cup G) - P(E \cap G)$  is the desired probability:



$P(E \Delta G)$  is shaded.

Thus  $P(E \Delta G) = P(E \cup G) - P(E \cap G) = 0.5$

5. A store sells three types of shoes: running shoes (40%), hiking shoes (30%) and raining shoes (30%). 10% of running shoes are water proof, 70% of hiking shoes are water proof, all raining shoes are, of course, water proof. If a selected shoe is water proof, what is the probability that it is a hiking shoe?

- A. 0.3/0.55  
 B. 0.21/0.55  
 C. 0.3/0.75  
 D. 0.21/0.91  
 E. 0.3/0.91

B

Want  $P(H|P)$

$P(R) = 0.4$      $P(H) = 0.3$      $P(W) = 0.3$

$P(P|R) = 0.1$      $P(P|H) = 0.7$      $P(P|W) = 1$

$$P(H|P) = \frac{P(P \cap H)}{P(P)}$$

$$P(P|H) = \frac{P(H \cap P)}{P(H)}$$

$$P(H \cap P) = P(P|H) \cdot P(H) / P(P)$$

$$P(P) = P(P|H) \cdot P(H) + P(P|R) \cdot P(R) + P(P|W) \cdot P(W)$$

$0.04$      $0.21$      $+ 0.3$

$0.55$

6. A plane has 300 seats. 60 of these are window seats. How many ways can we assign the seats for 200 passengers so that each window seat is occupied by a passenger?

- A.  $\frac{300! \cdot 200!}{(60!)^2 100!}$   
 B.  $\frac{200! \cdot 240!}{100! \cdot 140!}$   
 C.  $\frac{300! \cdot 240!}{140! \cdot 200!}$   
 D.  $\frac{200! \cdot 240!}{60! \cdot 140!}$   
 E.  $\frac{200! \cdot 240!}{(100!)^2 60!}$

$$\frac{240! \cdot 200!}{100! \cdot 140!}$$

$$\frac{240! \cdot 200!}{100! \cdot 140!}$$

300

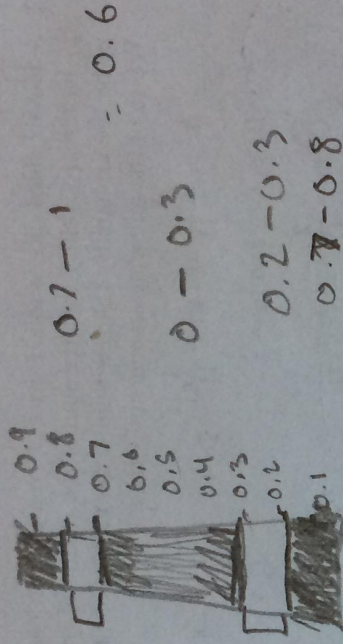
$210$      $200!$   
 $140$

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3. A 1 meter glass rod falls onto the floor and is broken into two pieces. Assume that the breaking point is uniform between 0 and 1 meter. Given that the longer piece has length at least 0.7 meter, what is the probability that the shorter piece has length at least 0.2 meter?

- A.  $\frac{0.1}{0.6}$   
 B.  $\frac{0.2}{0.6}$   
 C.  $\frac{0.3}{0.6}$   
 D.  $\frac{0.4}{0.6}$   
 E.  $\frac{0.5}{0.6}$

B



4. If  $A$ ,  $B$  and  $C$  are sets, then  $(A \cup B)^c \cup C^c =$

- ~~A.~~  $A^c \cup B^c \cup C^c$   
~~B.~~  $(A \cap C)^c \cup (B \cap C)^c$   
~~C.~~  $(A \cup C)^c \cup (B \cup C)^c$   
 D.  $(A \cap C)^c \cap (B \cap C)^c$   
~~E.~~  $(A \cup C)^c \cap (B \cup C)^c$

D

$$(A^c \cap B^c) \cup C^c$$

$$(A^c \cap B^c) \cup A^c \cap C^c \cup B^c \cap C^c$$

$$A^c \cup C^c \cap B^c$$

$$(A^c \cup C^c) \cap (B^c \cup C^c)$$

$$A^c \cap C^c \cap B^c \cup C^c$$

$$A^c \cup C^c \cap B^c \cup C^c$$

$$(A^c \cap B^c) \cup C^c$$

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$$(A \cup B)^c \cup C^c$$

Total score: 30 points Time: 50 minutes

Full name: Brandon Daugher (1 point)

## Part I: Multiple choice. 3 points each

1. A fair six sided dice is rolled twice. Given that at least one of the results is 3, let  $p_1$  be the probability that the sum of the results of the two rolls is 5; and  $p_2$  be the probability that the sum of the results of the two rolls is 6. Which of the following statements is true?

- A.  $p_1 = \frac{1}{11}$  and  $p_2 = \frac{1}{11}$   
 B.  $p_1 = \frac{1}{3}$  and  $p_2 = \frac{1}{6}$   
 C.  $p_1 = \frac{2}{11}$  and  $p_2 = \frac{1}{11}$   
 D.  $p_1 = \frac{1}{6}$  and  $p_2 = \frac{1}{6}$   
 E.  $p_1 = \frac{2}{11}$  and  $p_2 = \frac{2}{11}$

Handwritten notes for question 1:

3 1 1 3  
 5+3 2 2 → 5  
 p<sub>2</sub> → 3 3  
 3 4 4 3  
 3 5 5 3  
 3 6 6 3

2. How many ways can we assign 200 workers to 10 sites so that there are exactly 20 workers at each site?

- A.  $\frac{200!}{(20!)^{10}}$   
 B.  $\frac{200!}{(10!)^{20}}$   
 C.  $\frac{200!}{10!20!}$   
 D.  $10^{20}$   
 E.  $20^{10}$

Handwritten notes for question 2:

$200 \cdot \binom{180}{20} \cdot \binom{160}{20} \cdot \binom{140}{20}$   
 $20 \quad 20 \quad 20$   
 $120 \quad 100 \quad 80 \quad 60 \quad 40 \quad 20$   
 $20! \quad 20$

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