9

Total score: 10 points Time: 50 minutes 10 questions; 1 point each

Full name: JinJing Zhou (No points but must write)

1A ✓
2E ✗
3D ✓
4B ✓
5A ✓
6E ✓
7C ✓
8C ✓
9E ✓
10C ✓

1. Two fair six sided dice are rolled. What is the probability that the maximum of the two results is 4 given that the sum of the two results is 5 or 8?

   A. 1/3
   B. 2/7
   C. 3/7
   D. 2/9
   E. 4/9

4 1

4 4

$$\frac{\frac{3}{36}}{\frac{9}{36}} = \frac{1}{3}$$

	1	2	3	4	5	6
1				5		
2			5			8
3		5			8	
4	5			8		
5			8			
6		8				

2. For each natural number $n \geq 1$, let $A_n = \{n, n+2, n+4\}$. For what natural number $k \geq 1$, does the set $\bigcup_{n=k}^{\infty} A_n$ contain 5?

   A. 1, 2, 3 only.
   B. 1, 3, 5 only.
   C. 5 only.
   D. 1, 2, 3, 4, 5 only.
   E. None of the above

$A_1 = \{1, 3, 5\}$
$A_2 = \{2, 4, 6\}$

D 3. A box contains 3 blue balls, 1 red ball and 5 purple balls. 3 balls are selected from the box simultaneously. What is the probability that there are 1 blue ball and 2 purple balls?

A. 2/7

B. 3/7

C. 3/14

D. 5/14

E. 9/14

$\frac{1}{3}\cdot\frac{3}{9}\times\frac{5}{8}\times\frac{4}{7}$

$$\frac{\binom{3}{1}\cdot\binom{5}{2}}{9\times8\times7}$$

$\frac{3}{9}\times\frac{3}{8}\times\frac{4}{7}+\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}+\frac{5}{9}\times\frac{3}{8}\times\frac{4}{7}$

B 4. Let A,B,C be sets. Which of the following statements must be true?

$(A\cup B)\cap(A\cup C)$

I. If $B^c \subset A$, then $A\cup(B\cap C) = A\cup C$.

II. If $A\cap B \subset C$ then $A^c\cup B^c \subset C^c$.

III. $A\cap B^c = A\cap(A\cap B)^c$

$A\cup(C-B^c)$

$\Omega\cap(A\cup C)$

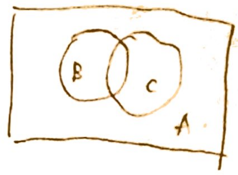
A. Only I and II are true.

B. Only I and III are true.

C. Only II and III are true.

D. Only III is true.

E. I, II and III are true.



A odd

B 1

C natural

$(A\cap B)^c = A^c\cup B^c$

$\cancel{AAA^c}$ $\emptyset\cup(A\cap B^c)$

$x\in C,\ x\notin \cancel{AB}\cup B$

$A^c\cup B^c=(A\cap B)^c$

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A 5. A box contains 4 blue balls and 6 red balls. 2 balls are selected from the box without replacement. Given that the second selected ball is blue, what is the probability that the first selected ball is red?

A. 2/3

B. 2/9

C. 4/9

D. 5/9

E. 15/24

P(first red second blue) = $\frac{6}{10} \times \frac{4}{9} = \frac{4}{15}$

P(first blue second blue) = $\frac{4}{10} \times \frac{3}{9} = \frac{2}{15}$

E 6. Let A and B be independent events. Assume that $\mathbb{P}(A) = 0.2$ and $\mathbb{P}(B) = 0.5$. Let $A\Delta B = (A \cap B^c) \cup (B \cap A^c)$. Find $\mathbb{P}(A\Delta B | A \cup B)$.

A. 1/6

B. 2/6

C. 3/6

D. 4/6

E. 5/6

independent ⇒ $P(A \cap B) = P(A) \cdot P(B)$
$= 0.1$

$P(A \Delta B) = 0.2 + 0.5 - 2P(A \cap B)$
$= 0.5$

0.1 0.1 0.4

$P(A \cup B) = 0.2 + 0.5 - 0.1$
$= 0.6$

C 7. A train has two compartments: Compartment 1 and Compartment 2. Compartment 1 has 20 seats and Compartment 2 has 30 seats. There will be 25 passengers taking the train. How many ways can we assign the passengers to the seats so that there are 5 passengers sitting in Compartment 1 and 20 passengers sitting in Compartment 2?

A. $\dfrac{20! \cdot 25!}{5! \cdot 10! \cdot 15!}$

B. $\dfrac{20! \cdot 25!}{10! \cdot 15! \cdot 30!}$

C. $\dfrac{25! \cdot 30!}{5! \cdot 10! \cdot 15!}$

D. $\dfrac{20! \cdot 30!}{5! \cdot 10! \cdot 15!}$

E. $\dfrac{20! \cdot 30!}{5! \cdot 15! \cdot 25!}$

Compart 1	Compartment 2
20	30
5	20

$$\binom{20}{5}\binom{30}{20} 5! \cdot 20! \cdot \binom{25}{5}$$

$$\frac{\cancel{20!}\, 30!\, \cancel{5!}\, \cancel{20!}\, 25!}{\cancel{5!}\, 15!\, \cancel{20!}\, 10!\, 5!\, \cancel{20!}}$$

$$\frac{25!\, 30!}{5!\, 10!\, 15!}$$

$$\binom{20}{5}\binom{30}{20}\binom{25}{5} \cdot 5!\, 20!$$

$$= \frac{\cancel{20!}\, 30!\, 25!\, \cancel{5!}\, \cancel{20!}}{5!\, 15!\, \cancel{20!}\, 10!\, \cancel{5!}\, \cancel{20!}}$$

C 8. Suppose that you and your friend will meet at a bus stop. Whoever arrives earlier will wait for the other one before taking the bus. Each of you and your friend will arrive at the bus stop some time between 3:00 pm and 3:20 pm with uniformly probability and independent of each other. Buses arrive at 3:05, 3:15, 3:25,.... What is the probability that you (and your friend) will take the 3:15 bus?

A. 100/400

B. 150/400

C. 200/400

D. 250/400

E. 300/400

~~later arrive~~

P(take 3:15 bus)

= P(both arrive before 3:15) − P(both arrive before 3:05)

$= \frac{15}{20} \times \frac{15}{20} - \frac{5}{20} \times \frac{5}{20}$

$= \frac{3}{4} \times \frac{3}{4} - \frac{1}{4} \times \frac{1}{4}$

$= \frac{1}{2}$

E 9. Let A and B be two events. If $\mathbb{P}(A^c|B) = 0.6$, $\mathbb{P}(B^c|A) = 0.2$ and $\mathbb{P}(A \cup B) = 0.55$, find $\mathbb{P}(B)$.

A. 0.3

B. 0.35

C. 0.4

D. 0.45

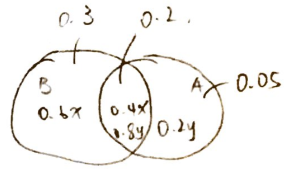
E. 0.5

$P(A^c|B)$

$= \frac{P(A^c \cap B)}{P(B)} = 0.6$

$\frac{P(B^c \cap A)}{P(A)} = 0.2$

$P(A \cup B) = 0.55$



$x + 0.2y = 0.55$

$0.6x + y = 0.55$

$5x + y = 2.75$

$0.6x + y = 0.55$

$4.4x = 2.2$

$x = 0.5$

$y = 0.25$

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C

10. In 1990, electric light bulbs were made at two plants:
Plant 1 (75%) and Plant 2 (25%).
In 1990, 20% of the defective bulbs were made at Plant 2.

$P(P2 \mid \text{defec}) = \frac{P(P2 \cap \text{defec})}{P(\text{defec})}$

$= \frac{0.06 \cdot 0.25}{0.08 \cdot 0.75 + 0.06 \cdot 0.25}$

In 1991, bulbs were still made at these two plants but different proportions:
Plant 1 (50%) and Plant 2 (50%).
In 1991, the overall defective rate was 7%.

$= \frac{0.015}{0.06 + 0.015}$

$= 20\%$

Assume that the defective rates of the bulbs made at each plant were the same for both years 1990 and 1991. Calculate the defective rate of the bulbs at Plant 1.

$P(\text{defec} \mid P1) = \frac{P(\text{defec} \cap P1)}{P(P1)}$

A. 7.25%

B. 7.5%

C. 8%

D. 8.5%

E. 9%

1990,

$P(P1) = 0.75 \quad P(P2) = 0.25$

$P(P2 \mid \text{defec}) = \frac{P(P2 \cap \text{defec})}{P(\text{defec})} = 0.2$

$P(\text{defec}) = P(\text{defec} \mid P1) \cdot P(P1) + P(\text{defec} \mid P2) \cdot P(P2)$

$\frac{P(\text{defec} \cap P2)}{P(\text{defec} \cap P1) + P(\text{defec} \cap P2)} = 0.2$

Let $P(\text{defec} \mid P2) = y$, $P(\text{defec} \mid P1) = x$

$\frac{y \cdot 0.25}{x \cdot 0.75 + y \cdot 0.25} = 0.2$

$0.25y = 0.15x + 0.05y$

$0.2y = 0.15x$

$y = 0.75x$

1991,

$P(P1) = 0.5 \quad P(P2) = 0.5$

$P(\text{defec}) = \frac{P(\text{defec} \mid P1) + P(\text{defec} \mid P2)}{2} = 0.07$

$x + y = 0.14$

$1.75x = 0.14$

$0.25x = 0.02$

$x = 0.08$

$y = 0.06$

End of exam