

## First Midterm Exam

Name:

1. **READ THE RULES FIRST**
2. Submit your answers to gradescope as our normal homework
3. Throughout the test, show your work so that your reasoning is clear. Otherwise no credit will be given.
4. Send me emails if you have questions. But in most cases, you will not get reply, please consider it as: **the exam is correct as it is.**
5. Help from other person, AI or **internet** are not allowed.
6. It will take about 1.5 hr to finish this exam.

Problem	Points	Grade
1	15	
2	20	
3	15	
4	15	
5	15	
6	15	
7	5	
Total	100	

1. (15 pts)

We roll lots of dice. Let

$$A_k = \{\text{The } k\text{-th die shows a 6}\} \quad k = 1, 2, \dots$$

**Using unions, intersections, complements to express each of the following events in terms of  $A_k$ 's.**

(a)  $B = \{\text{We have at least two sixes showing on the first three dice}\}$

(b)  $D = \{\text{We have exactly one 6 showing on the first 100 dice}\}$

2. (20 pts) Suppose that for events  $C$  and  $D$ , we know that  $P(C) = 0.4$  and  $P(D) = 0.3$ .
- (a) (2 pts) If  $C$  and  $D$  are disjoint, what is  $P(C \cup D)$ ?
  
  - (b) (2 pts) If  $C$  and  $D$  are independent, what is  $P(C \cup D)$ ?
  
  - (c) (2 pts) If  $C$  and  $D$  are disjoint, what is  $P(C|D)$ ?
  
  - (d) (2 pts) If  $C$  and  $D$  are independent, what is  $P(C|D)$ ?
  
  - (e) (2 pts) If  $D \subset C$  what is  $P(D|C)$ ?
  
  - (f) (2 pts) If  $D \subset C$  what is  $P(C|D)$ ?

(g) (2 pts)  $P(C \cap D) = 0.1$  what is  $P(C^c | D)$ ?

(h) (2 pts)  $P(C \cup D) = 0.5$  what is  $P(C^c | D)$ ?

(i) (2 pts)  $P(C|D) = 0.7$  what is  $P(C^c | D)$ ?

(j) (2 pts)  $P(D|C) = 0.7$  what is  $P(C^c | D)$ ?

Note: it is  $C^c$  in above 4 sub-questions.

3. (15 pts) Mr. Brown owns twelve pairs of shoes. One morning (while still being a bit sleepy) he grabs two shoes randomly.

(a) What is the probability that he will be able to wear the shoes he chose? (I.e. there is one right shoe and one left shoe.)

(b) What is the probability that he chose a matching pair of shoes?

4. (15 pts) You want to use the knowledge learnt (heard) in 170A to guess whether student  $A$  is in a relationship (i.e., has gf / bf). First you noticed (I don't know how) that 60% students in UCLA are in a relationship .

(a) Second, you find that 60% of the students who are not in relationship like to attend study groups, and this number is only 30% for the students who are in a relationship. You also find that student  $A$  likes to attend study group. Based on these information, what is the probability of

$\mathbb{P}(A \text{ is in a relationship})?$

(b) Furthermore, you find that among the students who are not in relationship and like to attend study group, 20% of them use smartphones more than 3 times/hour, and among the students who like to attend study group and in relationship, 50% of them use smartphones more than 3 times/hour. You also find that student  $A$  uses smartphone more than 3 times/hour. Now based on **all** of these information, what is the probability of

$\mathbb{P}(A \text{ is in a relationship})?$

5. (15pts) An urn has 10 black and 8 white balls that are randomly removed one at a time.

(a) (Without replacement) What is the probability that the 5th ball is white ?

(b) (Without replacement) What is the probability that there are 1 white and 4 black balls among the first 5 balls.

(c) Now if each time, we replaced a ball of the other color, i.e, (pick black ball replace white ball, pick white ball replace black ball), what is the probability that the first 5 balls are all white.

6. (15 points) A man possesses five fair coins, three of which are double-headed, and two are normal. He shuts his eyes, picks a coin at random, and tosses it.

(a) What is the probability that the lower face of the coin is a head?

(b) He opens his eyes and sees that the coin is showing heads; what is the new probability that the lower face is a head?

(c) He shuts his eyes again, and tosses the same coin again. Given all that the man has seen so far, now what is the probability that the lower face is a head?



7. (5pt)

Assume that  $A$ ,  $B$ ,  $C$  are independent events.

Is  $(A \cup B^c)$  independent of  $C$ ?

Please explain your reason.

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