Answer the questions in the spaces provided. Explain your answers clearly - if the answer requires computation, write it down. Unless otherwise noted, please simplify. No books, phones, or calculators are allowed - you may use your single page of notes. Sign your name below - by doing so you are agreeing to abide by the UCLA Student Code of Conduct.

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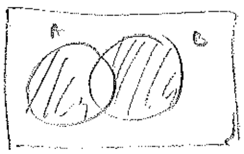
1. (15 points) Show the formula

$A \cap B$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$A \cup B \setminus A \cap B$

$$P((A \cap B^c) \cup (A^c \cap B)) = P(A) + P(B) - 2P(A \cap B).$$

Note: $A \cup B = (A \cap B^c) \cup (A^c \cap B) \cup (A \cap B)$



Proof $A \cap B^c$, $A^c \cap B$, $A \cap B$ are all disjoint

So $P((A \cap B^c) \cup (A^c \cap B) \cup (A \cap B)) =$

$P(A \cap B^c) + P(A^c \cap B) + P(A \cap B)$

Since $A \cup B = (A \cap B^c) \cup (A^c \cup B) \cup (A \cap B)$ and

$P(A \cup B) = P(A) + P(B) \quad P(A \cap B)$

$P(A) + P(B) - P(A \cap B) = P(A \cap B^c) + P(A^c \cap B) + P(A \cap B)$

Therefore, $P(A \cap B^c) + P(A^c \cup B) = P(A) + P(B) - 2P(A \cap B)$

and $P((A \cap B^c) \cup (A^c \cup B)) = P(A \vee B^c) + P(A^c \cup B)$

So $P((A \cap B^c) \cup (A^c \vee B)) = P(A) + P(B) - 2P(A \cap B)$

2. (15 points) Let A and B be events with $P(A) > 0$ and $P(B) > 0$. We say that an event B *suggests* an event A if $P(A|B) > P(A)$, and does not suggest event A if $P(A|B) < P(A)$.

(a) Show that B suggests A if and only if A suggests B.

Show iff $P(A|B) > P(A)$ then $P(B|A) > P(B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad : P(A \cap B) = P(B)P(A|B)$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \qquad P(A \cap B) = P(A)P(B|A)$$

$$P(A|B)P(B) = P(B|A)P(A)$$

$$\frac{P(A|B)}{P(A)} = \frac{P(B|A)}{P(B)}$$

if $P(A|B) > P(A)$, $\frac{P(A|B)}{P(A)} > 1$ so $\frac{P(B|A)}{P(B)} > 1$.

Therefore $P(B|A) > P(B)$

~~If~~ $P(B|A) > P(A)$, then $\frac{P(B|A)}{P(A)} \geq 1$ so

$P(A|B) > P(A)$

(b) Assume that $P(B^C) > 0$. Show that B suggests A if and only if B^C does not suggest A.

B^C does not suggest A

Show Iff $P(A|B^C) < P(A)$, then $P(A|B) > P(A)$

$$P(A|B^C) = \frac{P(A \cap B^C)}{P(B^C)} < P(A)$$

$$P(A \cap B^C) < P(A)P(B^C)$$

$$P(A \cap B^C) < P(A)(1 - P(B))$$

$$\underbrace{P(A \cap B^C)}_{< P(A)P(B^C)} + \underbrace{P(A \cap B)}_{> P(A)P(B)} = P(A)$$

$$P(A \cap B) > P(A)P(B)$$

3. (20 points) During a visit to a primary care physician's office, the probability of having neither lab work nor referral to a specialist is 0.33. Of those coming to that office, the probability of having lab work is 0.52 and the probability of having a referral is 0.41.

(a) What is the probability of having both lab work and a referral?

(1) $P(L^c \cap R^c) = 0.33 = P((L \cup R)^c)$

$P(L) = 0.52$

$P(R) = 0.41$

$P(R \cap L) = ?$

$L^c \cap R^c = (L \cup C)^c$ and (1)

$P((L \cup C)^c) = 1 - P(L \cup C)$

$P(L \cup C) = 1 - P((L \cup C)^c)$

$P(L \cup C) = 1 - 0.33 = 0.67$

$P(L \cap C) = P(L) + P(C) - P(L \cup C)$

$= 0.52 + 0.41 - 0.67$

$= 0.93 - 0.67$

0.26

$P(L \cap C) = \boxed{0.26}$

(b) Given that a patient is having lab work, what is the probability that they have a referral?

$P(R \mid L) = \frac{P(R \cap L)}{P(L)} = \frac{0.26 \text{ From (a)}}{0.52}$

$= \boxed{\frac{1}{2}}$

(c) Are having lab work and getting a referral independent events?

$P(L \cap R) = 0.26 \neq 0.52 \cdot 0.41 = P(L)P(R)$

also

$P(R \mid L) = \frac{1}{2} \neq P(R) = 0.41$

0.52×0.41

4. (10 points) A dish of hard candies contains 51 candies, of which 8 are white, 10 are tan, 7 are pink, 5 are purple, 6 are yellow, 8 are orange, and 7 are green. If you select nine pieces of candy randomly from the box, without replacement, give the probability that

(a) Exactly three of the candies are white. (You do not need to simplify.)

white, nonwhite

$p(\text{white}) = 8/51$

$p(W^c) = 43/51$

$$\begin{array}{r} 51 \\ -8 \\ \hline 43 \end{array}$$

$p = \frac{8}{51}$, binomial r.v

$$\boxed{\binom{9}{3}\left(\frac{8}{51}\right)^3\left(\frac{43}{51}\right)^6}$$

(b) Three are white, two are tan, one is pink, one is yellow, and two are green. (You do not need to simplify.)

✓ 2 tan out of 10 ✓ 3 white out of 8

✓ 1 pink out of 7

✓ 1 yellow out of 6

2 green out of 7

$$\frac{\binom{8}{3}\binom{10}{2}\binom{7}{1}\binom{6}{1}\binom{7}{2}}{\binom{51}{9}}$$

5. (10 points) A basketball player makes free throws with probability p. Assuming independence between two shots, can p be selected so that $P(\text{zero makes}) = P(\text{one make}) = P(\text{two makes})$? Explain why or why not clearly.

let k be # of made shots in binomial r.v

$P(\text{make}) = p$

$P(\text{zero makes}) = \binom{2}{0}(1-p)^2$ $\binom{2}{0} = \frac{2!}{2!}$

$P(1 \text{ make}) = \binom{2}{1}p(1-p)$

$P(2 \text{ makes}) = \binom{2}{2}p^2$

Show

$$\overset{(1)}{(1-p)^2} = \overset{(2)}{2(1-p)} = \overset{(3)}{p^2}$$

(1),(2) $1 - \cancel{2p} + p^2 = 2 - \cancel{2p}$

$1 + p^2 = 2$ $p = 1$

(1), (3) $1 - 2p^2 + \cancel{p^2} = \cancel{p^2}$

$1 - 2p^2 = 0$

$-2p^2 = -1$

$p^2 = \frac{1}{2}$ $p = \sqrt{\frac{1}{2}}$

(2),(3) $2(1-p) = p^2$

$2 - 2p = p^2$

$0 = p^2 + 2p - 2$

$p = 1 \rightarrow p^2 + 2p - 2 = 1 \neq 0$

$p = \sqrt{\frac{1}{2}} \rightarrow p^2 + 2p - 2 = \frac{1}{2} + \frac{2}{\sqrt{2}} - 2 \neq 0$

The three equations give conflicting values of p, all three cannot be satisfied

6. (15 points) It is believed that approximately 75% of American youth now have insurance due to the health care law. Suppose this is true, and let X equal the number of American youth in a random sample of $n = 16$ with health insurance.

(a) How is X distributed?

$$\mathbb{P}(X=x) = \begin{cases} 0.75, & x = 1 \\ 0.25, & x = 0 \end{cases}$$

$x = 1$ represents having health insurance, $X = 0$ represents not having insurance

(b) Find the probability that X is equal to 10. (You do not need to simplify.)

Y = binomial random variable

$$p(10) = P(Y=10) = \binom{16}{10}(0.75)^{10}(0.25)^{6}$$

(c) Suppose instead that a news organization is asking students on a college campus one by one if they have insurance. What is the probability that the first student who has insurance is the 10th student they ask? (You do not need to simplify.)

Z = geometric random variable

$$p(10) = P(Z=10) = \underbrace{(0.25)^9}_{\text{not having HI}}\underbrace{(0.75)^1}_{\text{having HI}}$$

7. (15 points) Bowl A contains three red and two white chips, and bowl B contains four red and three white chips. A chip is drawn at random from bowl A and transferred to bowl B. Compute the probability of then drawing a red chip from bowl B.

A: 3R 2W B: 4R 3W

transferred chip - $P(TW) = \frac{2}{5}$, $P(TW^c) = \frac{3}{5}$

Let TW be the event of transferring a white chip.

If you transfer white chip: 4R 4W

$P(R|TW) = \frac{4}{8} = \frac{1}{2}$

If you transfer red chip: 5R 3W

$P(R|TW) = \frac{5}{8}$

total prob. theorem

$$P(R) = P(R|TW)P(TW) + P(R|TW^c)P(TW^c)$$

$$= \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{5} + \frac{\cancel{5}}{8}\cdot\frac{3}{\cancel{5}}$$

$$= \frac{1}{5} + \frac{3}{8}$$

$$= \frac{8}{40} + \frac{15}{40} = \boxed{\frac{23}{40}}$$

15
+ 8
2