Math 170A - Midterm 1

Name: Jaclyn Stone

UID: 704963991

Problems	Points	Score
1	5	5
2	4	4
3	6	6
4	3	3
Total	18	18

$P(A \cup B \cup C) = P((A\cup B)\cup C)$, $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$P((A\cup B)\cup C) = P(A) + P(B) - P(A\cap B) + P(C) - P(A\cap B\cap C)$

5

Problem 1. *(5 points)*

a) (*1 points*) For Ω the sample space, $A, B \subseteq \Omega$ such that

$$A^c \cap B^c = \emptyset,$$

show that

$$A \cup B = \Omega.$$

b) (*1 points*) Compute the total number of anagrams of the word HONNOLD (note that an anagram in this case is any 7-letter word that can be formed from two O's, two N's, one H, one L and one D).

c) (*2 points*) In how many ways can we place n identical pawns in an $n \times n$ chessboard such that we never have two pawns in the same line or column? Justify your answer.

d) (*1 point*) Given that

$P(B|A) = \frac{P(B\cap A)}{P(A)}$

$$\mathbb{P}(A) = \frac{1}{3}, \quad \mathbb{P}(B|A) = \frac{1}{3}, \quad \mathbb{P}(C|A\cap B) = \frac{1}{3} = \frac{P(A\cap B\cap C)}{P(\cancel{A\cap} B\cap C)}$$

compute

$$\mathbb{P}(A\cap B\cap C).$$

a) By De'Morgan's $A^c \cap B^c = (A\cup B)^c = \emptyset$, $((A\cup B)^c)^c = \emptyset^c$,
$\emptyset^c = \underline{\Omega}$, $((A\cup B)^c)^c = A\cup B$, $\boxed{\text{so } A\cup B = \underline{\Omega}}$

b) Number of anagrams of word with n letters, k distinct letters, and $\{m_i\}_{i=1}^k$ of each distinct letter is $\frac{n!}{m_1!\cdot m_2!\cdot \ldots \cdot m_k!}$, for HONNOLD, $n=7$, $k=5$, $m_1 = 1$, $m_2 = 2$, $m_3 = 2$, $m_4 = 1$, $m_5 = 1$, so # of anagrams $: \frac{7!}{1!\,2!\,2!\,1!\,1!}$
$= \boxed{\frac{7!}{4}}$

c) Since n pawns, each column must have a pawn, start in first column, n choices of rows, next column has $n-1$ choices, then $n-2$ and so on, so # of configurations is $\boxed{n!}$

d) $P(A) = 1/3$, $P(B|A) = \frac{P(B\cap A)}{P(A)} = \frac{1}{3}$, so $P(B\cap A) = \frac{1}{9}$
$P(C|A\cap B) = \frac{P(A\cap B\cap C)}{P(A\cap B)} = \frac{1}{3}$, so $P(A\cap B\cap C) = \frac{1}{9}\cdot\frac{1}{3} = \frac{1}{27}$

$\boxed{\text{So } \mathbb{P}(A\cap B\cap C) = \frac{1}{27}}$

4

Problem 2. *(4 points)*

An experiment consists of three independent trials. The probability of success of the i-th trial is p_i for $i \in \{1, 2, 3\}$.

a) (*1 points*) Write down a sample space of the experiment.

b) (*1 point*) Write down the event that only one trial is successful.

c) (*2 points*) Compute the probability that at least two trials are successful.

a) An experiment can be a success (S) or failure (F)

So $\Omega = \{SSS, SSF, SFS, SFF, FSS, FSF, FFS, FFF\}$

b) P(one success) = ~~P({SFF, FSF, FFS})~~ $= P(\{SFF\} \cup \{FSF\} \cup \{FFS\})$

$= P(SFF) + P(FSF) + P(FFS)$ since disjoint, independent

$= \boxed{p_1(1-p_2)(1-p_2) + (1-p_1)p_2(1-p_3) + (1-p_1)(1-p_2)p_3}$

ok

c) P(at least two successes) $= P(\{SSS, SSF, SFS, FSS\})$

$= \boxed{p_1p_2p_3 + p_1p_2(1-p_3) + p_1(1-p_2)p_3 + (1-p_1)p_2p_3}$

note that we use independence of trials to say that P(SSS) = P(trial 1 success ∩ trial 2 success ∩ trial 3 success) = P(trial 1 success) · P(trial 2 success) · P(trial 3 success) $= p_1p_2p_3$

Scanned with CamScanner

Problem 3. *(6 points)*

An employee gets to work late 3 times out of 10 whenever she walks to the office, and 1 times out of 10 whenever she decides to order an uber. Given that she chooses to walk to the office 8 times out of 10 and order an uber 2 times of out 10, compute the following:

a) (*3 points*) The probability that she is late to work on a given day. Justify your answer.

b) (*3 points*) The probability that she walked to the office on a day that she was late. Justify your answer.

a) $P(W) = 8/10$, $P(W^c) = P(U) = 2/10$

$P(L \mid W) = 3/10$, $P(L \mid U) = 1/10$

$P(L) = P((L \cap W) \underbrace{\cup}_{\text{disjoint}} (L \cap W^c)) = P(L \cap W) + P(L \cap W^c)$ by axiom 2

$= P(W)P(L \mid W) + P(W^c)P(L \mid W^c)$ total probability

$= \frac{8}{10} \cdot \frac{3}{10} + \frac{2}{10} \cdot \frac{1}{10} = \frac{24}{100} + \frac{2}{100} = \frac{26}{100} = \boxed{26\% = \frac{13}{50}}$

b) Find $P(W \mid L)$, by baye's

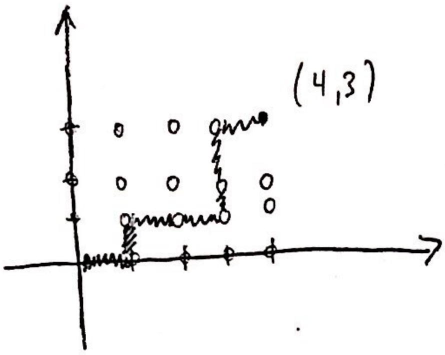
$P(W \mid L) = \frac{P(W)P(L \mid W)}{P(L \cap W) + P(L \cap W^c)} = \frac{P(W)P(L \mid W)}{P(L)} = \frac{\frac{8}{10} \cdot \frac{3}{10}}{\left(\frac{26}{100}\right)}$

$= \frac{\left(\frac{24}{100}\right)}{\left(\frac{26}{100}\right)} = \frac{24}{26} = \boxed{\frac{12}{13}}$

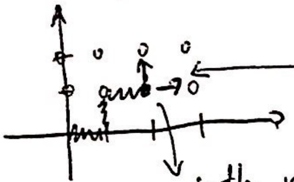
Scanned with CamScanner

Problem 4. *(3 points)*

For given numbers $m, n \in \mathbb{N}$ compute the total number of paths in $\mathbb{R}^2$ that start from $(0,0)$ and end up at (m,n) that consist of $m+n$ steps and every step is either in the direction of $(1,0)$ or $(0,1)$ and is always of length 1. Justify your answer.

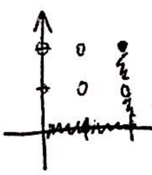


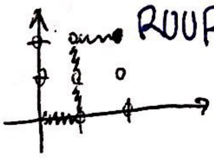
A possible path from $(0,0)$ to $(4,3)$

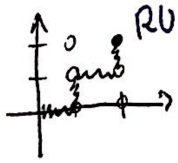


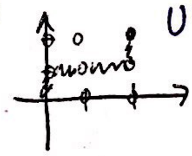
path reaches this point, $\boxed{\text{two options now}}$

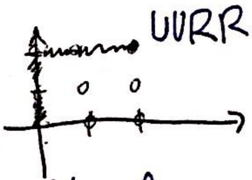
All paths from $(0,0)$ to $(2,2)$ (case $m=n=2$):

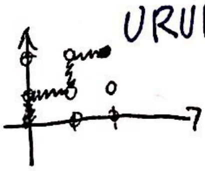

RRUU


RUUR


RURU


URRU


UURR


URUR

Note this is every anagram of RRUU, and confirm $\frac{4!}{2!2!} = 6$

Now in general, # of paths is # of anagrams of $\underbrace{R\cdots R}_{m}\underbrace{U\cdots U}_{n}$ so $\boxed{\frac{(m+n)!}{m!\,n!}}$