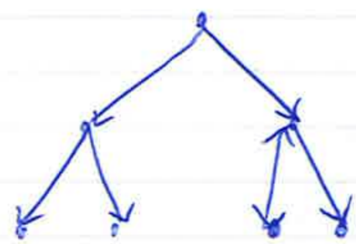
Week #8

Tues
5/19

Discussion

Midterm 2 Answers

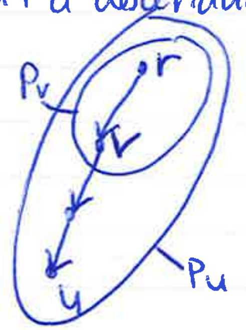
#1 Let $T=(V,E)$ be a directed tree. Prove that if u is a descendant of v, there is no path from u to v.



Proof/

Answer: Let P_v be the unique path from the root node of T to v, and let P_u be the unique path from the root node of T to u.

"u is a descendant of v" $\iff$ "P_v is a subgraph of P_u"



If I had a path from u to v, then I could get another path from r to v by going from r to u, then u to v. But, I know there's only one path from r to v, which is the desired contradiction.

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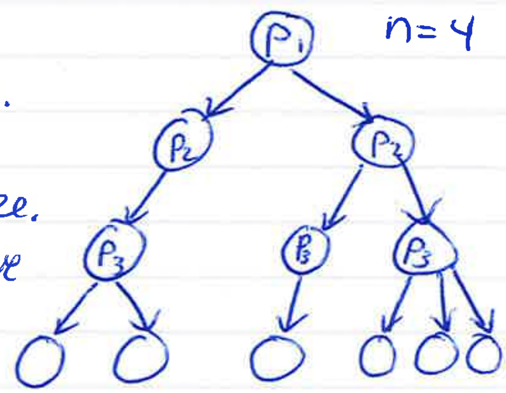
#5

Let $G = (T, P, S, V, M, \Pi, \mathcal{I})$ be a game with no chance. Let $\sigma_1, \ldots, \sigma_n$ be a set of strategies for players $1, \ldots, n$.
Prove that these strategies describe a unique path through the game tree.

Proof: Let n = longest path in tree be the number of turns in the game.
If $n = 1$, this is obviously true.
Assume it's true for all $k < n$, WTS true for n.



Look at the first $(n-1)$ levels of this game tree.
By the induction hypothesis, there exists unique path through the first $(n-1)$ levels of the game tree leading to a terminal node (in this subtree)
Then whoever moves next has a unique move determined by their strategy, leading to a unique terminal node in the full game tree.
By induction, true for all finite game trees. ☺ ✓