1. The strategy $q = (5/16, 7/16, 4/16)$ is optimal for Colin (player II) in the game

$$A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 2 \\ 2 & -1 & 0 \end{bmatrix}$$

(a) Calculate the value of the game. (b) Find the optimal strategy for Rose (player I).

2. If firm 1 produces q_1 units of a product and firm 2 produces q_2 units, the price $p(q_1, q_2)$ at which all $q_1 + q_2$ units will sell is $p(q_1, q_2) = 17 - q_1 - q_2$. The cost of production to firm 1 is 1 per unit plus a fixed cost of 2 and the cost to firm 2 is 3 per unit plus a fixed cost of 1. (a) Write the profit functions $u_1(q_1, q_2)$ and $u_2(q_1, q_2)$ for the two firms. (b) Show that if firm 1 produces q_1 units, then the optimal production for firm 2 is $q_2 = (14 - q_1)/2$ (Stackelberg model). (c) Find the equilibrium productions q_1 and q_2 for the two firms.

3. Find the MM-strategies and safety levels for the two players in the bimatrix game

$$(A, B) = \begin{bmatrix} (1,\ 4) & (4,\ 1) \\ (2,\ t) & (3,\ 3) \end{bmatrix}$$

for all real numbers t.

4. Suppose that $\hat{p} = (\hat{p}_1, \hat{p}_2, \hat{p}_3)$ is a strategy for Rose (player I) in a 3×3 matrix game A such that, against the pure strategies for Colin (player II), the expected payoffs are $\hat{p}^T A e_1 = 3, \hat{p}^T A e_2 = 2$ and $\hat{p}^T A e_3 = 5/2$. (a) Prove that $\hat{p}^T A q \geq 2$ for any mixed strategy $q = (q_1, q_2, q_3)$ for Colin. (b) Define the

lower value of the game A and use it, along with part (a), to prove that $V(A) \geq 2$, where $V(A)$ denotes the value of the game A.

5. (a) Solve the matrix game

$$\alpha = \begin{bmatrix} 0 & 7 \\ 9 & 2 \end{bmatrix}$$

(b) Use dominance to reduce the game

$$A = \begin{bmatrix} 0 & 7 & 4 \\ 8 & -3 & 7 \\ 4 & 4 & 3 \\ 9 & 2 & 5 \end{bmatrix}$$

to the game α of part (a), showing the dominances. (c) Write the optimal strategies and the value of the game A.